

ЗАДАНИЯ
к контрольным работам и типовым расчетам

1. Аффинные пространства

1.1. Задачи

Задача 1.1. Составить параметрические уравнения:

a) прямой, проходящей через точки A и B ;

b) плоскости, проходящей через точки A , B и C .

1. a) $A(9, 1, 7, -4)$, $B(1, 8, 2, 4)$;

b) $A(0, 9, -9, -3, 3)$, $B(-1, 4, -6, -7, -9)$, $C(-6, -8, 1, 3, -7)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = 9 - 8t \\ x_2 = 1 + 7t \\ x_3 = 7 - 5t \\ x_4 = -4 + 8t \end{array} \right. , \left\{ \begin{array}{l} x_1 = -t_1 - 6t_2 \\ x_2 = 9 - 5t_1 - 17t_2 \\ x_3 = -9 + 3t_1 + 10t_2 \\ x_4 = -3 - 4t_1 + 6t_2 \\ x_5 = 3 - 12t_1 - 10t_2 \end{array} \right.$$

2. a) $A(2, 0, -8, 0)$, $B(-5, -3, -2, 6)$;

b) $A(5, -10, 0, 0, -7)$, $B(-5, -8, 5, 4, 5)$, $C(-2, -9, -5, 10, 2)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = 2 - 7t \\ x_2 = -3t \\ x_3 = -8 + 6t \\ x_4 = 6t \end{array} \right. , \left\{ \begin{array}{l} x_1 = 5 - 10t_1 - 7t_2 \\ x_2 = -10 + 2t_1 + t_2 \\ x_3 = 5t_1 - 5t_2 \\ x_4 = 4t_1 + 10t_2 \\ x_5 = -7 + 12t_1 + 9t_2 \end{array} \right.$$

3. a) $A(-10, -4, 0, 3)$, $B(-7, 1, 8, -1)$;

b) $A(-5, 1, 10, 5, 4)$, $B(4, -3, 4, 2, -6)$, $C(4, 1, -9, 3, 4)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -10 + 3t \\ x_2 = -4 + 5t \\ x_3 = 8t \\ x_4 = 3 - 4t \end{array} \right. , \left\{ \begin{array}{l} x_1 = -5 + 9t_1 + 9t_2 \\ x_2 = 1 - 4t_1 \\ x_3 = 10 - 6t_1 - 19t_2 \\ x_4 = 5 - 3t_1 - 2t_2 \\ x_5 = 4 - 10t_1 \end{array} \right.$$

4. a) $A(0, -1, -4, -2)$, $B(6, -6, -8, 5)$;

b) $A(-8, 0, -8, -3, 6)$, $B(1, -5, -6, 0, 2)$, $C(3, 2, -10, -7, 0)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = 6t \\ x_2 = -1 - 5t \\ x_3 = -4 - 4t \\ x_4 = -2 + 7t \end{array} \right. , \left\{ \begin{array}{l} x_1 = -8 + 9t_1 + 11t_2 \\ x_2 = -5t_1 + 2t_2 \\ x_3 = -8 + 2t_1 - 2t_2 \\ x_4 = -3 + 3t_1 - 4t_2 \\ x_5 = 6 - 4t_1 - 6t_2 \end{array} \right.$$

5. a) $A(-10, -2, 5, -2)$, $B(8, -10, 7, -2)$;
 b) $A(-1, 3, -7, 0, -6)$, $B(-1, 3, 7, -3, 4)$, $C(0, -6, -7, -9, 5)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -10 + 18t \\ x_2 = -2 - 8t \\ x_3 = 5 + 2t \\ x_4 = -2 \end{array} \right. , \left\{ \begin{array}{l} x_1 = -1 + t_2 \\ x_2 = 3 - 9t_2 \\ x_3 = -7 + 14t_1 \\ x_4 = -3t_1 - 9t_2 \\ x_5 = -6 + 10t_1 + 11t_2 \end{array} \right.$$

6. a) $A(-10, -3, 3, -5)$, $B(1, -5, -1, 1)$;
 b) $A(0, 6, 5, 6, 1)$, $B(4, -8, -3, 7, -4)$, $C(3, 9, 1, -5, -3)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -10 + 11t \\ x_2 = -3 - 2t \\ x_3 = 3 - 4t \\ x_4 = -5 + 6t \end{array} \right. , \left\{ \begin{array}{l} x_1 = 4t_1 + 3t_2 \\ x_2 = 6 - 14t_1 + 3t_2 \\ x_3 = 5 - 8t_1 - 4t_2 \\ x_4 = 6 + t_1 - 11t_2 \\ x_5 = 1 - 5t_1 - 4t_2 \end{array} \right.$$

7. a) $A(-1, 8, 9, -9)$, $B(-4, 5, 1, 6)$;
 b) $A(6, 6, -9, 9, 3)$, $B(5, 10, -8, 0, -6)$, $C(-6, 2, 0, 9, -8)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -1 - 3t \\ x_2 = 8 - 3t \\ x_3 = 9 - 8t \\ x_4 = -9 + 15t \end{array} \right. , \left\{ \begin{array}{l} x_1 = 6 - t_1 - 12t_2 \\ x_2 = 6 + 4t_1 - 4t_2 \\ x_3 = -9 + t_1 + 9t_2 \\ x_4 = 9 - 9t_1 \\ x_5 = 3 - 9t_1 - 11t_2 \end{array} \right.$$

8. a) $A(-10, -8, -4, -8)$, $B(-4, -2, 9, 0)$;
 b) $A(7, -9, 6, -8, 8)$, $B(-10, 7, -8, 0, 6)$, $C(-3, -5, -10, 9, -8)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -10 + 6t \\ x_2 = -8 + 6t \\ x_3 = -4 + 13t \\ x_4 = -8 + 8t \end{array} \right. , \left\{ \begin{array}{l} x_1 = 7 - 17t_1 - 10t_2 \\ x_2 = -9 + 16t_1 + 4t_2 \\ x_3 = 6 - 14t_1 - 16t_2 \\ x_4 = -8 + 8t_1 + 17t_2 \\ x_5 = 8 - 2t_1 - 16t_2 \end{array} \right.$$

9. a) $A(6, 4, -1, -3)$, $B(-2, 7, 2, 7)$;
 b) $A(-2, 0, -2, 6, 1)$, $B(-3, 0, -3, -10, -1)$, $C(8, -3, -7, -7, 2)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = 6 - 8t \\ x_2 = 4 + 3t \\ x_3 = -1 + 3t \\ x_4 = -3 + 10t \end{array} \right. , \left\{ \begin{array}{l} x_1 = -2 - t_1 + 10t_2 \\ x_2 = -3t_2 \\ x_3 = -2 - t_1 - 5t_2 \\ x_4 = 6 - 16t_1 - 13t_2 \\ x_5 = 1 - 2t_1 + t_2 \end{array} \right.$$

10. a) $A(5, -3, -3, -10)$, $B(-9, -5, -5, -10)$;
 b) $A(0, 10, 7, -10, -4)$, $B(-1, 2, 10, -7, -7)$, $C(3, 3, 8, 6, 9)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = 5 - 14t \\ x_2 = -3 - 2t \\ x_3 = -3 - 2t \\ x_4 = -10 \end{cases}, \begin{cases} x_1 = -t_1 + 3t_2 \\ x_2 = 10 - 8t_1 - 7t_2 \\ x_3 = 7 + 3t_1 + t_2 \\ x_4 = -10 + 3t_1 + 16t_2 \\ x_5 = -4 - 3t_1 + 13t_2 \end{cases}$$

11. a) $A(-4, 5, 0, -3)$, $B(3, 8, -9, -3)$;
 b) $A(-7, 7, 5, -2, 10)$, $B(8, 0, 1, 9, 1)$, $C(-1, -4, 4, -4, 9)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -4 + 7t \\ x_2 = 5 + 3t \\ x_3 = -9t \\ x_4 = -3 \end{cases}, \begin{cases} x_1 = -7 + 15t_1 + 6t_2 \\ x_2 = 7 - 7t_1 - 11t_2 \\ x_3 = 5 - 4t_1 - t_2 \\ x_4 = -2 + 11t_1 - 2t_2 \\ x_5 = 10 - 9t_1 - t_2 \end{cases}$$

12. a) $A(-3, 3, 10, -3)$, $B(-7, -1, 6, -4)$;
 b) $A(-8, 0, 8, 2, 1)$, $B(1, -6, 1, -2, 2)$, $C(9, -2, 0, 5, 3)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -3 - 4t \\ x_2 = 3 - 4t \\ x_3 = 10 - 4t \\ x_4 = -3 - t \end{cases}, \begin{cases} x_1 = -8 + 9t_1 + 17t_2 \\ x_2 = -6t_1 - 2t_2 \\ x_3 = 8 - 7t_1 - 8t_2 \\ x_4 = 2 - 4t_1 + 3t_2 \\ x_5 = 1 + t_1 + 2t_2 \end{cases}$$

13. a) $A(-5, 7, 6, 8)$, $B(4, 2, -8, -7)$;
 b) $A(-1, 5, 9, 1, -3)$, $B(10, -1, 2, -2, 0)$, $C(0, 8, -4, 10, 10)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -5 + 9t \\ x_2 = 7 - 5t \\ x_3 = 6 - 14t \\ x_4 = 8 - 15t \end{cases}, \begin{cases} x_1 = -1 + 11t_1 + t_2 \\ x_2 = 5 - 6t_1 + 3t_2 \\ x_3 = 9 - 7t_1 - 13t_2 \\ x_4 = 1 - 3t_1 + 9t_2 \\ x_5 = -3 + 3t_1 + 13t_2 \end{cases}$$

14. a) $A(-7, -10, -8, 6)$, $B(-7, 0, -1, 1)$;
 b) $A(6, 3, 2, 3, -6)$, $B(-9, -4, -7, 2, -4)$, $C(-3, 1, 3, 5, 5)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -7 \\ x_2 = -10 + 10t \\ x_3 = -8 + 7t \\ x_4 = 6 - 5t \end{cases}, \begin{cases} x_1 = 6 - 15t_1 - 9t_2 \\ x_2 = 3 - 7t_1 - 2t_2 \\ x_3 = 2 - 9t_1 + t_2 \\ x_4 = 3 - t_1 + 2t_2 \\ x_5 = -6 + 2t_1 + 11t_2 \end{cases}$$

15. a) $A(-6, -4, -9, -5)$, $B(5, -4, 2, 10)$;
 b) $A(0, 3, -1, 4, -6)$, $B(-8, 7, 5, 6, -8)$, $C(-1, 3, -4, 0, -9)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -6 + 11t \\ x_2 = -4 \\ x_3 = -9 + 11t \\ x_4 = -5 + 15t \end{array} \right\}, \left\{ \begin{array}{l} x_1 = -8t_1 - t_2 \\ x_2 = 3 + 4t_1 \\ x_3 = -1 + 6t_1 - 3t_2 \\ x_4 = 4 + 2t_1 - 4t_2 \\ x_5 = -6 - 2t_1 - 3t_2 \end{array} \right.$$

16. a) $A(-1, -6, 9, -5)$, $B(10, 9, -9, 0)$;
 b) $A(10, 9, 4, -2, 5)$, $B(10, 4, 2, -8, 2)$, $C(3, 2, -6, -7, 8)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -1 + 11t \\ x_2 = -6 + 15t \\ x_3 = 9 - 18t \\ x_4 = -5 + 5t \end{array} \right\}, \left\{ \begin{array}{l} x_1 = 10 - 7t_2 \\ x_2 = 9 - 5t_1 - 7t_2 \\ x_3 = 4 - 2t_1 - 10t_2 \\ x_4 = -2 - 6t_1 - 5t_2 \\ x_5 = 5 - 3t_1 + 3t_2 \end{array} \right.$$

17. a) $A(-6, 6, 5, -2)$, $B(-3, -7, -3, -7)$;
 b) $A(-4, -7, -3, 1, 7)$, $B(7, 8, 3, -8, -6)$, $C(4, 4, -1, 7, 2)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -6 + 3t \\ x_2 = 6 - 13t \\ x_3 = 5 - 8t \\ x_4 = -2 - 5t \end{array} \right\}, \left\{ \begin{array}{l} x_1 = -4 + 11t_1 + 8t_2 \\ x_2 = -7 + 15t_1 + 11t_2 \\ x_3 = -3 + 6t_1 + 2t_2 \\ x_4 = 1 - 9t_1 + 6t_2 \\ x_5 = 7 - 13t_1 - 5t_2 \end{array} \right.$$

18. a) $A(-1, 9, -8, 1)$, $B(-5, -5, 3, 8)$;
 b) $A(-9, 6, 9, 2, -5)$, $B(4, -3, -5, 5, 2)$, $C(6, 3, -1, -4, 4)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -1 - 4t \\ x_2 = 9 - 14t \\ x_3 = -8 + 11t \\ x_4 = 1 + 7t \end{array} \right\}, \left\{ \begin{array}{l} x_1 = -9 + 13t_1 + 15t_2 \\ x_2 = 6 - 9t_1 - 3t_2 \\ x_3 = 9 - 14t_1 - 10t_2 \\ x_4 = 2 + 3t_1 - 6t_2 \\ x_5 = -5 + 7t_1 + 9t_2 \end{array} \right.$$

19. a) $A(-1, 7, 0, -8)$, $B(4, 9, 6, -10)$;
 b) $A(-8, -10, 3, -6, -4)$, $B(6, -2, 8, 5, 1)$, $C(-9, -6, -4, -8, 3)$.

$$\text{Ответ: } \left\{ \begin{array}{l} x_1 = -1 + 5t \\ x_2 = 7 + 2t \\ x_3 = 6t \\ x_4 = -8 - 2t \end{array} \right\}, \left\{ \begin{array}{l} x_1 = -8 + 14t_1 - t_2 \\ x_2 = -10 + 8t_1 + 4t_2 \\ x_3 = 3 + 5t_1 - 7t_2 \\ x_4 = -6 + 11t_1 - 2t_2 \\ x_5 = -4 + 5t_1 + 7t_2 \end{array} \right.$$

20. a) $A(8, 4, 8, 0)$, $B(7, 10, 6, -2)$;
 b) $A(3, -2, 1, -1, -8)$, $B(2, 1, -3, -5, 5)$, $C(10, 5, -9, 0, 8)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = 8 - t \\ x_2 = 4 + 6t \\ x_3 = 8 - 2t \\ x_4 = -2t \end{cases}, \begin{cases} x_1 = 3 - t_1 + 7t_2 \\ x_2 = -2 + 3t_1 + 7t_2 \\ x_3 = 1 - 4t_1 - 10t_2 \\ x_4 = -1 - 4t_1 + t_2 \\ x_5 = -8 + 13t_1 + 16t_2 \end{cases}$$

21. a) $A(-9, 6, -9, -4)$, $B(5, 7, -2, 4)$;
 b) $A(2, 5, 3, 7, 7)$, $B(-7, 1, -7, 0, 2)$, $C(-1, -5, -3, 10, 5)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -9 + 14t \\ x_2 = 6 + t \\ x_3 = -9 + 7t \\ x_4 = -4 + 8t \end{cases}, \begin{cases} x_1 = 2 - 9t_1 - 3t_2 \\ x_2 = 5 - 4t_1 - 10t_2 \\ x_3 = 3 - 10t_1 - 6t_2 \\ x_4 = 7 - 7t_1 + 3t_2 \\ x_5 = 7 - 5t_1 - 2t_2 \end{cases}$$

22. a) $A(10, -7, 8, 9)$, $B(-10, 10, 7, -4)$;
 b) $A(-7, 1, 5, 7, 4)$, $B(8, 8, -7, -6, 5)$, $C(0, -2, 6, -10, -9)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = 10 - 20t \\ x_2 = -7 + 17t \\ x_3 = 8 - t \\ x_4 = 9 - 13t \end{cases}, \begin{cases} x_1 = -7 + 15t_1 + 7t_2 \\ x_2 = 1 + 7t_1 - 3t_2 \\ x_3 = 5 - 12t_1 + t_2 \\ x_4 = 7 - 13t_1 - 17t_2 \\ x_5 = 4 + t_1 - 13t_2 \end{cases}$$

23. a) $A(-9, -7, 0, -4)$, $B(8, 10, 4, 9)$;
 b) $A(7, 8, 0, -3, -1)$, $B(-1, 2, -9, 7, -6)$, $C(-10, -4, -3, 8, 0)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -9 + 17t \\ x_2 = -7 + 17t \\ x_3 = 4t \\ x_4 = -4 + 13t \end{cases}, \begin{cases} x_1 = 7 - 8t_1 - 17t_2 \\ x_2 = 8 - 6t_1 - 12t_2 \\ x_3 = -9t_1 - 3t_2 \\ x_4 = -3 + 10t_1 + 11t_2 \\ x_5 = -1 - 5t_1 + t_2 \end{cases}$$

24. a) $A(-7, -6, -10, 1)$, $B(2, 3, 7, 3)$;
 b) $A(-6, 8, 2, -3, 2)$, $B(-10, -2, 9, 5, 5)$, $C(10, 9, 4, 0, 6)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -7 + 9t \\ x_2 = -6 + 9t \\ x_3 = -10 + 17t \\ x_4 = 1 + 2t \end{cases}, \begin{cases} x_1 = -6 - 4t_1 + 16t_2 \\ x_2 = 8 - 10t_1 + t_2 \\ x_3 = 2 + 7t_1 + 2t_2 \\ x_4 = -3 + 8t_1 + 3t_2 \\ x_5 = 2 + 3t_1 + 4t_2 \end{cases}$$

25. a) $A(-7, -8, -1, 8)$, $B(-2, -9, 5, 10)$;
 b) $A(0, -8, 2, 10, 0)$, $B(6, 8, 3, -9, -2)$, $C(8, 4, -1, -10, 1)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -7 + 5t \\ x_2 = -8 - t \\ x_3 = -1 + 6t \\ x_4 = 8 + 2t \end{cases}, \begin{cases} x_1 = 6t_1 + 8t_2 \\ x_2 = -8 + 16t_1 + 12t_2 \\ x_3 = 2 + t_1 - 3t_2 \\ x_4 = 10 - 19t_1 - 20t_2 \\ x_5 = -2t_1 + t_2 \end{cases}$$

26. a) $A(8, 0, 2, -5)$, $B(7, -2, -4, 9)$;
 b) $A(-2, 3, 1, 0, -2)$, $B(0, -8, 3, -3, -9)$, $C(-8, 8, 4, -9, 7)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = 8 - t \\ x_2 = -2t \\ x_3 = 2 - 6t \\ x_4 = -5 + 14t \end{cases}, \begin{cases} x_1 = -2 + 2t_1 - 6t_2 \\ x_2 = 3 - 11t_1 + 5t_2 \\ x_3 = 1 + 2t_1 + 3t_2 \\ x_4 = -3t_1 - 9t_2 \\ x_5 = -2 - 7t_1 + 9t_2 \end{cases}$$

27. a) $A(6, 6, 4, 10)$, $B(5, -7, -10, -1)$;
 b) $A(-8, 0, 7, -1, 9)$, $B(6, 4, 9, -9, 6)$, $C(10, -2, 2, 8, -4)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = 6 - t \\ x_2 = 6 - 13t \\ x_3 = 4 - 14t \\ x_4 = 10 - 11t \end{cases}, \begin{cases} x_1 = -8 + 14t_1 + 18t_2 \\ x_2 = 4t_1 - 2t_2 \\ x_3 = 7 + 2t_1 - 5t_2 \\ x_4 = -1 - 8t_1 + 9t_2 \\ x_5 = 9 - 3t_1 - 13t_2 \end{cases}$$

28. a) $A(-9, 3, 3, 6)$, $B(-6, -10, 2, -4)$;
 b) $A(0, -5, 4, -10, -5)$, $B(6, -4, 10, -1, -5)$, $C(6, 2, -8, 8, -4)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -9 + 3t \\ x_2 = 3 - 13t \\ x_3 = 3 - t \\ x_4 = 6 - 10t \end{cases}, \begin{cases} x_1 = 6t_1 + 6t_2 \\ x_2 = -5 + t_1 + 7t_2 \\ x_3 = 4 + 6t_1 - 12t_2 \\ x_4 = -10 + 9t_1 + 18t_2 \\ x_5 = -5 + t_2 \end{cases}$$

29. a) $A(-4, -1, -8, -3)$, $B(8, 1, 10, -6)$;
 b) $A(7, -5, 6, 3, -5)$, $B(2, -8, -10, -6, 9)$, $C(-4, -5, -4, 3, 4)$.

$$\text{ОТВЕТ: } \begin{cases} x_1 = -4 + 12t \\ x_2 = -1 + 2t \\ x_3 = -8 + 18t \\ x_4 = -3 - 3t \end{cases}, \begin{cases} x_1 = 7 - 5t_1 - 11t_2 \\ x_2 = -5 - 3t_1 \\ x_3 = 6 - 16t_1 - 10t_2 \\ x_4 = 3 - 9t_1 \\ x_5 = -5 + 14t_1 + 9t_2 \end{cases}$$

30. a) $A(-7, -5, -10, 9)$, $B(6, 2, 3, 1)$;
 b) $A(-9, -1, 6, -8, -7)$, $B(2, 8, 6, -1, 4)$, $C(0, 8, -6, -1, -3)$.

$$\text{Ответ: } \begin{cases} x_1 = -7 + 13t \\ x_2 = -5 + 7t \\ x_3 = -10 + 13t \\ x_4 = 9 - 8t \end{cases}, \begin{cases} x_1 = -9 + 11t_1 + 9t_2 \\ x_2 = -1 + 9t_1 + 9t_2 \\ x_3 = 6 - 12t_2 \\ x_4 = -8 + 7t_1 + 7t_2 \\ x_5 = -7 + 11t_1 + 4t_2 \end{cases}$$

Задача 1.2. Найти уравнение гиперплоскости, проходящей через точки A , B , C и D .

1. $A(-1, 2, 3, -1)$, $B(3, 2, 2, 0)$,
 $C(1, 0, 3, -1)$, $D(1, 2, -3, 3)$.

Ответ: $x_1 + x_2 - 7x_3 - 11x_4 + 9 = 0$

2. $A(-3, -1, -2, 1)$, $B(-3, -1, 2, -3)$,
 $C(-2, 3, 3, 3)$, $D(1, -1, -2, -1)$.

Ответ: $4x_1 - 15x_2 + 8x_3 + 8x_4 + 5 = 0$

3. $A(-3, -3, 0, 1)$, $B(0, -2, -1, 0)$,
 $C(-1, -2, -1, 1)$, $D(0, -3, 1, -2)$.

Ответ: $x_1 - 2x_2 + x_4 - 4 = 0$

4. $A(-2, -1, -2, 3)$, $B(-1, 3, -3, -1)$,
 $C(2, -1, -3, 1)$, $D(-3, 2, -2, -1)$.

Ответ: $2x_1 + 2x_2 + 6x_3 + x_4 + 15 = 0$

5. $A(1, -1, -3, -3)$, $B(3, -2, 2, 2)$,
 $C(-3, 1, -3, 1)$, $D(3, -2, 1, -1)$.

Ответ: $x_1 + 2x_2 + 1 = 0$

6. $A(-2, 2, 0, 3)$, $B(3, -3, 1, 2)$,
 $C(-3, 3, 1, -2)$, $D(-2, 2, -3, 1)$.

Ответ: $x_1 + x_2 = 0$

7. $A(2, -1, -1, 3)$, $B(3, -1, 2, -2)$,
 $C(-1, -1, -1, 0)$, $D(1, 3, 1, -2)$.

Ответ: $x_1 - 2x_3 - x_4 - 1 = 0$

8. $A(-1, 1, -2, -1), \quad B(-1, 1, 1, -1),$
 $C(0, -1, 1, -1), \quad D(3, 1, 1, -2).$

Ответ: $2x_1 + x_2 + 8x_4 + 9 = 0$

9. $A(2, 0, 0, 3), \quad B(2, -1, -1, -3),$
 $C(1, 0, 1, 3), \quad D(0, 3, -1, 1).$

Ответ: $10x_1 + 8x_2 + 10x_3 - 3x_4 - 11 = 0$

10. $A(-2, 1, -3, 3), \quad B(1, -1, 1, 1),$
 $C(1, -2, 1, 2), \quad D(0, 3, -3, 2).$

Ответ: $4x_1 - 8x_2 - 11x_3 - 8x_4 + 7 = 0$

11. $A(-1, -3, 2, -3), \quad B(-3, 2, -2, 0),$
 $C(3, -3, -2, -3), \quad D(3, -2, -2, -2).$

Ответ: $x_1 + 3x_2 + x_3 - 3x_4 - 1 = 0$

12. $A(1, 3, 2, 2), \quad B(-1, 0, 0, 2),$
 $C(-3, -1, 2, 2), \quad D(-3, -1, 2, 1).$

Ответ: $2x_1 - 2x_2 + x_3 + 2 = 0$

13. $A(3, 1, -1, -2), \quad B(2, -2, 2, 3),$
 $C(0, -3, 1, 2), \quad D(2, -3, -3, 2).$

Ответ: $2x_1 - 3x_2 + x_3 - 2x_4 - 6 = 0$

14. $A(2, 1, -2, -2), \quad B(1, 2, -3, 1),$
 $C(2, 2, -1, -3), \quad D(2, 0, -1, -3).$

Ответ: $2x_1 + x_3 + x_4 = 0$

15. $A(3, 1, -3, -2), \quad B(-2, 1, 1, -1),$
 $C(3, 3, -1, -3), \quad D(-1, 2, 0, -3).$

Ответ: $8x_1 - 19x_2 + 13x_3 - 12x_4 + 10 = 0$

16. $A(2, -3, -2, -1), \quad B(-2, 2, -3, 1),$
 $C(1, -2, -3, -2), \quad D(0, 0, 2, 3).$

Ответ: $7x_1 + 6x_2 - x_4 + 3 = 0$

17. $A(-1, 2, 3, -1), \quad B(-1, 0, -3, 2),$
 $C(-3, 1, -2, 1), \quad D(-3, 2, 0, 0).$

Ответ: $x_1 - 2x_3 - 4x_4 + 3 = 0$

18. $A(-1, 0, 1, -2), \quad B(3, 0, 0, -2),$
 $C(0, -1, 3, 2), \quad D(2, 0, 1, 1).$

Ответ: $x_1 + 5x_2 + 4x_3 - x_4 - 5 = 0$

19. $A(1, -2, 2, 1), \quad B(-1, 2, 1, 0),$
 $C(3, -2, 0, -2), \quad D(-3, -2, -3, -1).$

Ответ: $11x_1 + 6x_2 - 16x_3 + 18x_4 + 15 = 0$

20. $A(-1, -2, 0, 2), \quad B(0, -2, -3, 2),$
 $C(0, -3, 2, -3), \quad D(-1, -1, 1, 2).$

Ответ: $15x_1 - 5x_2 + 5x_3 + 6x_4 - 7 = 0$

21. $A(1, -1, 1, -1), \quad B(1, 3, -2, 0),$
 $C(-2, 2, 2, 1), \quad D(-3, -1, 3, 1).$

Ответ: $4x_1 - x_2 + x_3 + 7x_4 + 1 = 0$

22. $A(-2, -1, 2, 1), \quad B(2, 0, 2, 3),$
 $C(3, 1, -1, -3), \quad D(3, 0, 3, 1).$

Ответ: $x_1 - 4x_2 - x_3 = 0$

23. $A(3, 1, 1, 2), \quad B(1, 1, -2, 2),$
 $C(-1, -1, 1, 1), \quad D(0, 0, 2, 0).$

Ответ: $9x_1 - 13x_2 - 6x_3 - 10x_4 + 12 = 0$

24. $A(0, -1, 0, -1), \quad B(3, -3, 0, -3),$
 $C(-3, -2, 1, -1), \quad D(3, 0, 1, -2).$

Ответ: $2x_1 - 3x_2 + 3x_3 + 6x_4 + 3 = 0$

25. $A(1, -3, 2, 1), \quad B(2, -1, 2, 1),$
 $C(-1, -1, -1, 3), \quad D(2, 1, 0, -3).$

Ответ: $16x_1 - 8x_2 - 14x_3 + 3x_4 - 15 = 0$

26. $A(-2, 1, 1, -3), \quad B(1, -1, -1, -3),$
 $C(-2, 0, 1, -2), \quad D(-2, 3, -1, 1).$

Ответ: $8x_1 + 3x_2 + 9x_3 + 3x_4 + 13 = 0$

27. $A(0, 2, -1, 1), \quad B(1, 3, -3, 1),$
 $C(0, 3, 2, -1), \quad D(0, -2, -1, 0).$

Ответ: $7x_1 - x_2 + 3x_3 + 4x_4 + 1 = 0$

28. $A(3, -2, 0, -3), \quad B(1, 3, 3, -2),$
 $C(-2, -3, 2, 3), \quad D(0, 1, 2, -1).$

Ответ: $2x_1 + 2x_2 - 3x_3 + 3x_4 + 7 = 0$

29. $A(-1, 1, 3, 3), \quad B(3, 3, -3, 3),$
 $C(3, -1, 3, -2), \quad D(0, 2, -3, 3).$

Ответ: $3x_1 - 9x_2 - x_3 + 6x_4 - 3 = 0$

30. $A(1, 3, 0, 0), \quad B(-3, -1, 0, 2),$
 $C(1, -1, -2, 3), \quad D(3, -2, -3, 2).$

Ответ: $4x_1 - 3x_2 + 9x_3 + 2x_4 + 5 = 0$

Задача 1.3. Найти систему линейных уравнений, задающую плоскость \mathcal{P} .

$$1. \mathcal{P} : \begin{cases} x^1 = -1 + t_1, \\ x^2 = t_1 + t_2, \\ x^3 = -t_1 - 2t_2, \\ x^4 = -3 - 2t_1 + 2t_2, \\ x^5 = -2 - 2t_1 - t_2. \end{cases}$$

Ответ: $-x^1 + 2x^2 + x^3 - 1 = 0, 4x^1 - 2x^2 + x^4 + 7 = 0, x^1 + x^2 + x^5 + 3 = 0$

$$2. \mathcal{P} : \begin{cases} x^1 = 2 + 3t_1 + 2t_2 + t_3, \\ x^2 = -2 - 2t_1 - 2t_2, \\ x^3 = -2 - 2t_2 + t_3, \\ x^4 = 2 + 3t_1 + 2t_2 + 3t_3, \\ x^5 = 2t_1 - t_2. \end{cases}$$

Ответ: $-5x^1 - 6x^2 + 2x^3 + x^4 = 0, 3x^1 + 11/2x^2 - 3x^3 + x^5 - 1 = 0$

$$3. \mathcal{P} : \begin{cases} x^1 = -1 - 2t_1 + t_2, \\ x^2 = -3 + t_1 - t_2, \\ x^3 = 2, \\ x^4 = -1 + 3t_1 - t_2, \\ x^5 = 3 + 2t_1. \end{cases}$$

Ответ: $x^3 - 2 = 0, 2x^1 + x^2 + x^4 + 6 = 0, 2x^1 + 2x^2 + x^5 + 5 = 0$

$$4. \mathcal{P} : \begin{cases} x^1 = 3t_2 + 2t_3, \\ x^2 = 1 - t_1 - t_2 + 2t_3, \\ x^3 = 2 - 2t_1 - t_2, \\ x^4 = 2 + 3t_1 - t_2 - t_3, \\ x^5 = -2 + 2t_1 - t_2. \end{cases}$$

Ответ: $\frac{11}{14}x^1 - 2/7x^2 + \frac{23}{14}x^3 + x^4 - 5 = 0, 4/7x^1 - 4/7x^2 + \frac{9}{7}x^3 + x^5 = 0$

$$5. \mathcal{P} : \begin{cases} x^1 = -1 - 3t_1 - 2t_2, \\ x^2 = -2 - 2t_1, \\ x^3 = -3 + 2t_1 + 2t_2, \\ x^4 = 2 + 2t_1 + t_2, \\ x^5 = 1 + t_1 + 2t_2. \end{cases}$$

Ответ: $x^1 - 1/2 x^2 + x^3 + 3 = 0, 1/2 x^1 + 1/4 x^2 + x^4 - 1 = 0, x^1 - x^2 + x^5 - 2 = 0$

$$6. \mathcal{P} : \begin{cases} x^1 = -3 - t_1 + 2t_2 + t_3, \\ x^2 = 2t_2 + t_3, \\ x^3 = -2t_1 + t_2 - 2t_3, \\ x^4 = 3 + 3t_1 - t_2 + t_3, \\ x^5 = -2t_1 + 2t_2 - 2t_3. \end{cases}$$

Ответ: $9/5 x^1 - 8/5 x^2 + 3/5 x^3 + x^4 + \frac{12}{5} = 0, 2/5 x^1 - 4/5 x^2 - 6/5 x^3 + x^5 + 6/5 = 0$

$$7. \mathcal{P} : \begin{cases} x^1 = -1 + 2t_2, \\ x^2 = 2t_1, \\ x^3 = t_1 - 2t_2, \\ x^4 = -2 + 3t_1 - t_2, \\ x^5 = 3 - 3t_2. \end{cases}$$

Ответ: $x^1 - 1/2 x^2 + x^3 + 1 = 0, 1/2 x^1 - 3/2 x^2 + x^4 + 5/2 = 0, 3/2 x^1 + x^5 - 3/2 = 0$

$$8. \mathcal{P} : \begin{cases} x^1 = 3 + 2t_1 + 3t_2 - 2t_3, \\ x^2 = -3 + t_1 + t_2 - t_3, \\ x^3 = -2 + t_2 - 2t_3, \\ x^4 = 1 - t_1 - t_2 - 2t_3, \\ x^5 = 3 - t_1 + t_2 - t_3. \end{cases}$$

Ответ: $3/2 x^1 - 2x^2 - 3/2 x^3 + x^4 - \frac{29}{2} = 0, -x^1 + 3x^2 - x^3 + x^5 + 7 = 0$

$$9. \mathcal{P} : \begin{cases} x^1 = 2 - 3t_1 + 2t_2, \\ x^2 = -2 + 2t_1 - 3t_2, \\ x^3 = 2 - 3t_1 - 2t_2, \\ x^4 = -3 - t_1 - 3t_2, \\ x^5 = -1 - 2t_1. \end{cases}$$

$$\text{ОТВЕТ: } -\frac{13}{5}x^1 - \frac{12}{5}x^2 + x^3 - 8/5 = 0, -9/5x^1 - \frac{11}{5}x^2 + x^4 + \frac{11}{5} = 0, -6/5x^1 - 4/5x^2 + x^5 + 9/5 = 0$$

$$10. \mathcal{P} : \begin{cases} x^1 = 1 + 3t_1 - 3t_3, \\ x^2 = 2 - 2t_1 + 2t_2 + t_3, \\ x^3 = -3 + t_2 + 2t_3, \\ x^4 = 2 - 2t_1 + 2t_2 + 3t_3, \\ x^5 = 1 - 3t_1 + 2t_2. \end{cases}$$

$$\text{ОТВЕТ: } \frac{4}{15}x^1 - 3/5x^2 - 4/5x^3 + x^4 - \frac{52}{15} = 0, 1/15x^1 - 7/5x^2 + 4/5x^3 + x^5 + \frac{62}{15} = 0$$

$$11. \mathcal{P} : \begin{cases} x^1 = -1 + 2t_1 + 2t_2, \\ x^2 = 1 - 3t_1 + 3t_2, \\ x^3 = 1 + t_2, \\ x^4 = 1 + 3t_1 - 2t_2, \\ x^5 = -1 + t_1 - 3t_2. \end{cases}$$

$$\text{ОТВЕТ: } -1/4x^1 - 1/6x^2 + x^3 - \frac{13}{12} = 0, -1/4x^1 + 5/6x^2 + x^4 - \frac{25}{12} = 0, 1/2x^1 + 2/3x^2 + x^5 + 5/6 = 0$$

$$12. \mathcal{P} : \begin{cases} x^1 = -3t_1 + t_2 + 3t_3, \\ x^2 = 3 + t_1 - 3t_2 - t_3, \\ x^3 = -2t_1 + t_2 - t_3, \\ x^4 = -2 - t_2 - 3t_3, \\ x^5 = 2 + 3t_1 + t_3. \end{cases}$$

$$\text{ОТВЕТ: } 1/2x^1 - 1/2x^2 - x^3 + x^4 + 7/2 = 0, \frac{7}{24}x^1 + \frac{13}{24}x^2 + 4/3x^3 + x^5 - \frac{29}{8} = 0$$

$$13. \mathcal{P} : \begin{cases} x^1 = -2 + 2t_1 + t_2, \\ x^2 = 3 - 2t_2 - t_3, \\ x^3 = 2t_1 + 3t_2 - t_3, \\ x^4 = 3 + 2t_1 + 2t_2 - t_3, \\ x^5 = -2 - t_2 + t_3. \end{cases}$$

Ответ: $-1/4 x^1 - 1/4 x^2 - 3/4 x^3 + x^4 - 11/4 = 0, -3/4 x^1 + 1/4 x^2 + 3/4 x^3 + x^5 - 1/4 = 0$

$$14. \mathcal{P} : \begin{cases} x^1 = 3 - 2t_1 + t_2, \\ x^2 = 2 - t_1 + 3t_2, \\ x^3 = -1 + 2t_1, \\ x^4 = -1 + 3t_1 + 2t_2, \\ x^5 = 2t_1 - t_2. \end{cases}$$

Ответ: $6/5 x^1 - 2/5 x^2 + x^3 - 9/5 = 0, \frac{11}{5} x^1 - 7/5 x^2 + x^4 - \frac{14}{5} = 0, x^1 + x^5 - 3 = 0$

$$15. \mathcal{P} : \begin{cases} x^1 = 2t_2 + t_3, \\ x^2 = -2 + 3t_1 - 3t_3, \\ x^3 = 2 + 2t_1 - 3t_2 + 3t_3, \\ x^4 = -1 - 3t_1 + 3t_2, \\ x^5 = -3t_1 - 3t_2. \end{cases}$$

Ответ: $-\frac{6}{13} x^1 + \frac{7}{13} x^2 + \frac{9}{13} x^3 + x^4 + \frac{9}{13} = 0, \frac{24}{13} x^1 + \frac{11}{13} x^2 + 3/13 x^3 + x^5 + \frac{16}{13} = 0$

$$16. \mathcal{P} : \begin{cases} x^1 = 2 + 3t_1 + t_2, \\ x^2 = -t_1 + 3t_2, \\ x^3 = 2 - t_1 - 2t_2, \\ x^4 = 3 - 3t_1 - 3t_2, \\ x^5 = -3 + 3t_1 + 2t_2. \end{cases}$$

Ответ: $1/2 x^1 + 1/2 x^2 + x^3 - 3 = 0, 6/5 x^1 + 3/5 x^2 + x^4 - \frac{27}{5} = 0, -\frac{11}{10} x^1 - 3/10 x^2 + x^5 + \frac{26}{5} = 0$

$$17. \mathcal{P} : \begin{cases} x^1 = 2t_1 + 2t_3, \\ x^2 = -t_1 + t_2 - 2t_3, \\ x^3 = 2 - t_1 + 2t_2 + t_3, \\ x^4 = 1 + t_1 + t_2 + 3t_3, \\ x^5 = -1 - t_1 + 3t_2. \end{cases}$$

$$\text{ОТВЕТ: } -5/8 x^1 + 1/2 x^2 - 3/4 x^3 + x^4 + 1/2 = 0, -1/2 x^1 - x^2 - x^3 + x^5 + 3 = 0$$

$$18. \mathcal{P} : \begin{cases} x^1 = -2 + 2t_1 + 2t_2, \\ x^2 = 1 + 2t_1, \\ x^3 = 3 - t_1, \\ x^4 = -2t_1 - t_2, \\ x^5 = -1 + 2t_1 - t_2. \end{cases}$$

$$\text{ОТВЕТ: } 1/2 x^2 + x^3 - 7/2 = 0, 1/2 x^1 + 1/2 x^2 + x^4 + 1/2 = 0, 1/2 x^1 - 3/2 x^2 + x^5 + 7/2 = 0$$

$$19. \mathcal{P} : \begin{cases} x^1 = 2 - 2t_1 + 2t_2 + 3t_3, \\ x^2 = 3 - 3t_1 - 3t_2, \\ x^3 = -2 - t_1 - t_2 + 2t_3, \\ x^4 = -2 + 2t_1 + t_2 - t_3, \\ x^5 = 1 + 3t_1 - 3t_2 + t_3. \end{cases}$$

$$\text{ОТВЕТ: } 1/4 x^1 + \frac{11}{24} x^2 + 1/8 x^3 + x^4 + 3/8 = 0, 3/2 x^1 + \frac{11}{12} x^2 - 11/4 x^3 + x^5 - \frac{49}{4} = 0$$

$$20. \mathcal{P} : \begin{cases} x^1 = -2 + 3t_1 + t_2, \\ x^2 = -3 - 2t_1 + t_2, \\ x^3 = 2 + 2t_1 + 3t_2, \\ x^4 = 1 + t_1 - t_2, \\ x^5 = 1 - t_1 + 2t_2. \end{cases}$$

$$\text{ОТВЕТ: } -8/5 x^1 - 7/5 x^2 + x^3 - \frac{47}{5} = 0, 1/5 x^1 + 4/5 x^2 + x^4 + 9/5 = 0, -3/5 x^1 - 7/5 x^2 + x^5 - \frac{32}{5} = 0$$

$$21. \mathcal{P} : \begin{cases} x^1 = 1 + 3t_2, \\ x^2 = -1 + t_1 + t_2, \\ x^3 = -3 + t_1 - 3t_2 + 2t_3, \\ x^4 = -2 + 2t_1 - 2t_2 - t_3, \\ x^5 = 1 + 3t_1 - 3t_2. \end{cases}$$

Ответ: $2x^1 - 5/2x^2 + 1/2x^3 + x^4 - 1 = 0, 2x^1 - 3x^2 + x^5 - 6 = 0$

$$22. \mathcal{P} : \begin{cases} x^1 = 2 - 3t_1 + t_2 + 2t_3, \\ x^2 = 2 + 2t_1 - t_2 - 2t_3, \\ x^3 = 3 - 2t_1 + 3t_2 - 3t_3, \\ x^4 = t_1 + 2t_2 - 2t_3, \\ x^5 = -2 - t_1 + t_2 + t_3. \end{cases}$$

Ответ: $7/3x^1 + 7/3x^2 - 2/3x^3 + x^4 - \frac{22}{3} = 0, 5/9x^1 + \frac{11}{9}x^2 - 1/9x^3 + x^5 - \frac{11}{9} = 0$

$$23. \mathcal{P} : \begin{cases} x^1 = 1 + t_2, \\ x^2 = -2 - t_1 - t_2, \\ x^3 = 2 - 2t_1 + 2t_2, \\ x^4 = 2 - t_1, \\ x^5 = t_1 + 3t_2. \end{cases}$$

Ответ: $-4x^1 - 2x^2 + x^3 - 2 = 0, -x^1 - x^2 + x^4 - 3 = 0, -2x^1 + x^2 + x^5 + 4 = 0$

$$24. \mathcal{P} : \begin{cases} x^1 = t_1 + 3t_2 + t_3, \\ x^2 = -1 - 3t_2 + 3t_3, \\ x^3 = 1 + 2t_1 + 3t_2 - 2t_3, \\ x^4 = -1 - t_1 + 2t_2 - 3t_3, \\ x^5 = -1 + 3t_1 + t_2. \end{cases}$$

Ответ: $1/7x^1 + \frac{26}{21}x^2 + 3/7x^3 + x^4 + \frac{38}{21} = 0, 1/7x^1 - \frac{23}{21}x^2 - \frac{11}{7}x^3 + x^5 + \frac{31}{21} = 0$

$$25. \mathcal{P} : \begin{cases} x^1 = -1 - 3t_1 + 2t_2, \\ x^2 = -2t_1 + 2t_2, \\ x^3 = -3 + 3t_1 - 2t_2, \\ x^4 = 1 + 3t_1 + 2t_2, \\ x^5 = -1 - 3t_1. \end{cases}$$

Ответ: $x^1 + x^3 + 4 = 0, 5x^1 - 6x^2 + x^4 + 4 = 0, -3x^1 + 3x^2 + x^5 - 2 = 0$

$$26. \mathcal{P} : \begin{cases} x^1 = -1 - 3t_1 - 3t_3, \\ x^2 = -2 - t_1 - t_2 - t_3, \\ x^3 = -3 - t_2 + t_3, \\ x^4 = -1 - 2t_2 + t_3, \\ x^5 = 3 - 3t_1 - t_2 + 3t_3. \end{cases}$$

Ответ: $1/3 x^1 - x^2 - x^3 + x^4 - 11/3 = 0, -8/3 x^1 + 5x^2 - 6x^3 + x^5 - \frac{41}{3} = 0$

$$27. \mathcal{P} : \begin{cases} x^1 = t_1 - 2t_2, \\ x^2 = 3 + 2t_1 + t_2, \\ x^3 = 1 + t_2, \\ x^4 = -1 + 2t_1, \\ x^5 = -1 - 2t_1 + 2t_2. \end{cases}$$

Ответ: $2/5 x^1 - 1/5 x^2 + x^3 - 2/5 = 0, -2/5 x^1 - 4/5 x^2 + x^4 + \frac{17}{5} = 0, 6/5 x^1 + 2/5 x^2 + x^5 - 1/5 = 0$

$$28. \mathcal{P} : \begin{cases} x^1 = 2 + 3t_1 - 2t_2 - 2t_3, \\ x^2 = -1 + 2t_1 - t_2 + 2t_3, \\ x^3 = -t_2 + 2t_3, \\ x^4 = -3 - 3t_1 - 3t_2 - t_3, \\ x^5 = -3 + 2t_2 + t_3. \end{cases}$$

Ответ: $-7/6 x^1 + \frac{13}{4} x^2 - \frac{47}{12} x^3 + x^4 + \frac{103}{12} = 0, 5/6 x^1 - 5/4 x^2 + \frac{19}{12} x^3 + x^5 + 1/12 = 0$

$$29. \mathcal{P} : \begin{cases} x^1 = -2 - 3t_1 - 2t_2, \\ x^2 = 2 - t_1 + t_2, \\ x^3 = -3 + 2t_1 - t_2, \\ x^4 = 2 + 2t_2, \\ x^5 = 1 - t_1 - t_2. \end{cases}$$

Ответ: $1/5 x^1 + 7/5 x^2 + x^3 + 3/5 = 0, 2/5 x^1 - 6/5 x^2 + x^4 + 6/5 = 0, -2/5 x^1 + 1/5 x^2 + x^5 - \frac{11}{5} = 0$

$$30. \mathcal{P} : \begin{cases} x^1 = -1 - t_1 + t_2 - t_3, \\ x^2 = 3 - 2t_1 - 2t_2 - t_3, \\ x^3 = -2t_1 - 2t_3, \\ x^4 = 2 - t_1 - 3t_2 - 3t_3, \\ x^5 = 1 + 2t_1 - t_2 + t_3. \end{cases}$$

Ответ: $7x^1 + 2x^2 - 6x^3 + x^4 - 1 = 0, 3x^1 + x^2 - 3/2x^3 + x^5 - 1 = 0$

Задача 1.4. Определить размерность плоскости, заданной системой линейных уравнений. Составить параметрические уравнения этой плоскости.

$$1. \begin{cases} -9x^1 - 2x^2 - 5x^3 - 7x^4 + 3x^5 = -5, \\ 4x^2 - 10x^3 + 15x^4 - 6x^5 = -10, \\ -4x^1 + 8x^2 + 9x^3 - 8x^4 - 12x^5 = 9, \\ -7x^1 + 2x^2 + 9x^3 - 17x^4 - 3x^5 = 9. \end{cases}$$

Ответ: $x^1 = -\frac{17}{14}t_1, x^2 = -\frac{25}{28}t_1 + 3/2t_2, x^3 = \frac{8}{7}t_1 + 1, x^4 = t_1, x^5 = t_2$

$$2. \begin{cases} -x^1 - 6x^2 - 3x^3 - 10x^4 + 4x^5 = -4, \\ -8x^1 - 9x^2 + 15x^3 + 14x^4 - 8x^5 = 4, \\ -5x^1 - 4x^2 - 2x^3 - 11x^4 + 8x^5 = -3, \\ 20x^1 + 15x^2 - 4x^3 - x^4 + 4x^5 = -1. \end{cases}$$

Ответ: $x^1 = 3/13t - \frac{28}{143}, x^2 = -\frac{9}{13}t + \frac{32}{143}, x^3 = -\frac{9}{13}t + \frac{6}{143}, x^4 = t + 3/11, x^5 = t$

$$3. \begin{cases} -10x^1 + 3x^2 - 3x^3 + 8x^4 - 4x^5 = -12, \\ -11x^1 + 4x^2 - 3x^3 + 15x^4 + 5x^5 = 0, \\ 5x^1 - 13x^2 + 8x^3 - 3x^4 + 19x^5 = 12, \\ x^2 - x^3 - 4x^4 - 8x^5 = -9. \end{cases}$$

Ответ: $x^1 = 2t_1 + 2t_2 - 3/2$, $x^2 = -5t_1 - 7t_2 + 21/2$, $x^3 = -9t_1 - 15t_2 + \frac{39}{2}$, $x^4 = t_1$, $x^5 = t_2$

$$4. \begin{cases} -6x^1 + 6x^3 - 15x^4 - 9x^5 = 15, \\ -2x^1 - 8x^2 + 10x^3 - 3x^4 - x^5 = 7, \\ -6x^1 + 6x^3 - 15x^4 - 9x^5 = 15, \\ -3x^1 - 2x^2 + 5x^3 - 7x^4 - 4x^5 = 8. \end{cases}$$

Ответ: $x^1 = t_1 - 5/2t_2 - 3/2t_3 - 5/2$, $x^2 = t_1 + 1/4t_2 + 1/4t_3 - 1/4$, $x^3 = t_1$, $x^4 = t_2$, $x^5 = t_3$

$$5. \begin{cases} 4x^1 - 4x^2 + 4x^3 - 7x^4 - x^5 = -3, \\ 2x^1 - 12x^2 + 8x^3 - 14x^4 = -7, \\ -x^1 + 9x^2 - 10x^3 + 13x^4 = 5, \\ -5x^1 + x^2 + 10x^3 - 4x^4 + x^5 = 2. \end{cases}$$

Ответ: $x^1 = 3/7t_1 + 2/7t_2 - 3/14$, $x^2 = -\frac{9}{14}t_1 + 1/14t_2 + 4/7$, $x^3 = \frac{19}{28}t_1 + 1/28t_2 + 1/28$, $x^4 = t_1$, $x^5 = t_2$

$$6. \begin{cases} 10x^1 + 22x^2 + 16x^3 + 4x^4 - x^5 = -4, \\ -3x^1 - x^2 + 2x^3 - 15x^4 + 11x^5 = -9, \\ 8x^1 + 16x^3 + 4x^4 - 6x^5 = -4, \\ x^1 + 3x^2 - 6x^3 - 11x^4 + 7x^5 = -13. \end{cases}$$

Ответ: $x^1 = \frac{31}{58}t - \frac{44}{29}$, $x^2 = -\frac{8}{29}t + \frac{4}{29}$, $x^3 = -\frac{3}{58}t + \frac{8}{29}$, $x^4 = \frac{37}{58}t + \frac{27}{29}$, $x^5 = t$

$$7. \begin{cases} 8x^1 - 6x^2 - 4x^3 - 4x^5 = -3, \\ -5x^1 + 3x^2 + 14x^3 - 10x^4 + 3x^5 = 1, \\ -19x^1 - 21x^2 + 2x^3 + 7x^4 = -1, \\ 20x^1 + 18x^2 - 4x^3 - 8x^4 = -1. \end{cases}$$

Ответ: $x^1 = 1/2t, x^2 = -1/4t + 1/4, x^3 = 3/8t + 3/8, x^4 = 1/2t + 1/2, x^5 = t$

$$8. \begin{cases} 21x^1 + 2x^2 - 6x^3 + 9x^4 = -12, \\ -27x^1 - x^2 + 5x^3 - 9x^4 + 2x^5 = 11, \\ -9x^1 + 8x^2 - 3x^3 - 4x^4 - 4x^5 = -1, \\ -6x^1 - x^2 - 4x^3 + 5x^4 + 10x^5 = -6. \end{cases}$$

Ответ: $x^1 = 2/7t + 1/21, x^2 = \frac{12}{7}t - 5/7, x^3 = 2t, x^4 = 2/7t - \frac{9}{7}, x^5 = t$

$$9. \begin{cases} -2x^1 + 6x^2 + 2x^3 - 3x^4 + 9x^5 = 1, \\ -9x^1 - 10x^2 - 6x^3 - x^4 - 8x^5 = -1, \\ -3x^1 + 12x^3 + 18x^4 - 9x^5 = 6, \\ -4x^1 - 4x^2 + 7x^3 + 14x^4 - 11x^5 = 4. \end{cases}$$

Ответ: $x^1 = -2/13t_1 + \frac{5}{13}t_2 - 2/13, x^2 = \frac{25}{26}t_1 - \frac{43}{26}t_2 - 1/26, x^3 = -\frac{20}{13}t_1 + \frac{11}{13}t_2 + \frac{6}{13}, x^4 = t_1, x^5 = t_2$

$$10. \begin{cases} -12x^1 - 2x^2 + 7x^3 - 5x^4 - 16x^5 = 6, \\ -11x^1 + 4x^2 + 13x^3 - 2x^4 - 5x^5 = 3, \\ 11x^1 - x^2 - 14x^3 + 11x^4 + 19x^5 = -10, \\ -6x^1 - 6x^2 - x^3 + 3x^4 - 8x^5 = -1. \end{cases}$$

Ответ: $x^1 = -4/7t - 3/14, x^2 = -4/3t, x^3 = 1/7t - 1/14, x^4 = -\frac{23}{21}t - \frac{11}{14}, x^5 = t$

$$11. \begin{cases} 15x^1 + 6x^2 - x^3 - 5x^4 + 5x^5 = -3, \\ 15x^1 + 4x^2 + x^3 = -2, \\ 3x^1 + 4x^2 - 3x^3 - 8x^4 + 8x^5 = -2, \\ -2x^2 + 2x^3 + 5x^4 - 5x^5 = 1. \end{cases}$$

Ответ: $x^1 = -1/3t_1 - 2/3t_2 + 2/3t_3, x^2 = t_1 + 5/2t_2 - 5/2t_3 - 1/2, x^3 = t_1, x^4 = t_2, x^5 = t_3$

$$12. \begin{cases} -6x^1 - 18x^2 + 6x^3 + 12x^4 + 9x^5 = 11, \\ 15x^1 + 8x^2 + 4x^3 - 2x^4 - 17x^5 = -6, \\ 5x^1 - 8x^2 + 12x^3 + 10x^4 - 3x^5 = 6, \\ 9x^1 + 5x^2 + 13x^3 + 4x^4 - 8x^5 = 0. \end{cases}$$

Ответ: $x^1 = 7/6 t_2 - \frac{5}{18}, x^2 = 1/2 t_1 + 1/24 t_2 - \frac{29}{72}, x^3 = -1/2 t_1 - \frac{5}{24} t_2 + \frac{25}{72}, x^4 = t_1, x^5 = t_2$

$$13. \begin{cases} -15x^1 + 15x^2 - 10x^3 + 9x^4 - 7x^5 = -6, \\ 5x^1 + 10x^2 - 4x^3 + 13x^4 - 12x^5 = 6, \\ -9x^1 + 7x^2 - 4x^3 + 23x^4 - x^5 = 0, \\ -4x^1 + 9x^2 - 5x^3 + 2x^4 + 4x^5 = 12. \end{cases}$$

Ответ: $x^1 = 14/3 t - 9/2, x^2 = -\frac{41}{3} t + 18, x^3 = -27 t + 33, x^4 = 4/3 t - 3/2, x^5 = t$

$$14. \begin{cases} -x^1 - 5x^2 + 15x^3 + 3x^4 - 9x^5 = 16, \\ -3x^1 + 2x^2 + 11x^3 + 12x^4 - 16x^5 = 6, \\ -3x^1 - 6x^2 + 10x^3 - 6x^5 = 14, \\ -8x^1 - 7x^3 + 4x^4 - 8x^5 = -2. \end{cases}$$

Ответ: $x^1 = 1/32 t + \frac{9}{32}, x^2 = -\frac{85}{64} t - \frac{125}{64}, x^3 = -3/16 t + \frac{5}{16}, x^4 = \frac{111}{64} t + \frac{39}{64}, x^5 = t$

$$15. \begin{cases} 4x^1 + 12x^2 + 5x^3 + 4x^4 + 12x^5 = 6, \\ -4x^1 + 12x^2 - x^3 - 4x^4 + 12x^5 = -6, \\ 6x^1 - 3x^2 + 4x^3 + 6x^4 - 3x^5 = 9, \\ 8x^1 + 6x^3 + 8x^4 = 12. \end{cases}$$

Ответ: $x^1 = -3/4 t_1 - t_2 + 3/2, x^2 = -1/6 t_1 - t_3, x^3 = t_1, x^4 = t_2, x^5 = t_3$

$$16. \begin{cases} 6x^2 - 2x^3 - 8x^4 - 4x^5 = 6, \\ 3x^1 + 4x^2 - x^3 - 6x^4 - 4x^5 = 5, \\ 15x^1 - 4x^2 + 3x^3 + 2x^4 - 4x^5 = 1, \\ 9x^1 - 3x^2 + 2x^3 + 2x^4 - 2x^5 = 0. \end{cases}$$

Ответ: $x^1 = -1/9 t_1 + 2/9 t_2 + 4/9 t_3 + 1/3, x^2 = 1/3 t_1 + 4/3 t_2 + 2/3 t_3 + 1, x^3 = t_1, x^4 = t_2, x^5 = t_3$

$$17. \begin{cases} -13x^1 - 5x^2 + 2x^3 + x^4 + 16x^5 = 7, \\ -9x^1 + 8x^2 - 19x^3 - 11x^4 + 6x^5 = -8, \\ -4x^1 - 12x^2 + 5x^3 + 11x^4 + 6x^5 = 7, \\ -5x^1 + 17x^2 + 11x^3 + 2x^4 + 7x^5 = -3. \end{cases}$$

Ответ: $x^1 = 7/6 t - \frac{5}{18}, x^2 = 1/12 t - \frac{97}{180}, x^3 = -1/4 t + \frac{29}{60}, x^4 = 1/12 t - \frac{49}{180}, x^5 = t$

$$18. \begin{cases} -7x^1 + 9x^2 + 5x^3 + 2x^4 - 19x^5 = 6, \\ x^1 - 4x^2 - 7x^3 + 22x^4 - 5x^5 = -5, \\ -7x^1 - 17x^2 - 15x^3 + 18x^4 + 9x^5 = -12, \\ 6x^1 + 11x^2 + 11x^3 - 20x^4 = 8. \end{cases}$$

Ответ: $x^1 = -\frac{187}{229} t - \frac{34}{229}, x^2 = \frac{206}{229} t - \frac{25}{229}, x^3 = \frac{176}{229} t + \frac{261}{229}, x^4 = \frac{154}{229} t + \frac{28}{229}, x^5 = t$

$$19. \begin{cases} -7x^1 + 3x^2 - 5x^3 - 4x^4 + 7x^5 = -1, \\ -4x^1 - 9x^2 - 5x^3 + 2x^4 + 4x^5 = 3, \\ 13x^1 + 3x^2 + 11x^3 + 4x^4 - 13x^5 = -1, \\ -10x^1 - 8x^3 - 4x^4 + 10x^5 = 0. \end{cases}$$

Ответ: $x^1 = -4/5 t_1 - 2/5 t_2 + t_3, x^2 = -1/5 t_1 + 2/5 t_2 - 1/3, x^3 = t_1, x^4 = t_2, x^5 = t_3$

$$20. \begin{cases} 11x^1 + 8x^2 + 15x^3 + 8x^4 - 8x^5 = -4, \\ -x^1 + 8x^2 + 3x^3 - 8x^4 + 8x^5 = -4, \\ 13x^1 + 9x^2 + 18x^3 + 7x^4 - 8x^5 = -7, \\ 4x^1 - 7x^2 + 3x^3 - 3x^4 - 2x^5 = -9. \end{cases}$$

Ответ: $x^1 = -7 t_1 + 5 t_2 + 5, x^2 = -2 t_1 + t_2 + 2, x^3 = \frac{17}{3} t_1 - 11/3 t_2 - 5, x^4 = t_1, x^5 = t_2$

$$21. \begin{cases} x^1 + 9x^2 - 12x^3 + 2x^4 + 4x^5 = -6, \\ 6x^1 - 6x^2 + 4x^3 + 13x^4 + x^5 = -11, \\ 2x^1 - 6x^2 + 8x^3 + 3x^4 - 5x^5 = -1, \\ 4x^1 + 3x^2 - 9x^3 + 11x^4 + 10x^5 = -12. \end{cases}$$

Ответ: $x^1 = -\frac{13}{8}t_1 + \frac{7}{8}t_2 - \frac{15}{8}, x^2 = \frac{9}{8}t_1 + \frac{21}{8}t_2 + 3/8, x^3 = \frac{7}{8}t_1 + \frac{19}{8}t_2 + 5/8, x^4 = t_1, x^5 = t_2$

$$22. \begin{cases} 6x^1 - 14x^2 - 9x^3 + 2x^4 + 6x^5 = 6, \\ 12x^1 + x^2 - 7x^3 - 9x^4 + 4x^5 = 0, \\ 17x^1 - 12x^2 - 17x^3 + 4x^4 + 11x^5 = 7, \\ 9x^1 + 2x^2 - 4x^3 - 6x^4 + 3x^5 = -5. \end{cases}$$

Ответ: $x^1 = -4/7t - \frac{23}{7}, x^2 = 1/4t + 1, x^3 = -1/7t - \frac{32}{7}, x^4 = -\frac{5}{28}t - 5/7, x^5 = t$

$$23. \begin{cases} -11x^1 + 2x^2 + 12x^3 - 7x^4 - 13x^5 = 7, \\ 5x^1 + 3x^2 + 5x^3 - 12x^4 - 7x^5 = 8, \\ -5x^2 - 13x^3 + 19x^4 + 16x^5 = -14, \\ -8x^1 + 6x^4 + 2x^5 = -3. \end{cases}$$

Ответ: $x^1 = 3/4t_1 + 1/4t_2 + 3/8, x^2 = \frac{7}{8}t_1 - 3/8t_2 + \frac{11}{16}, x^3 = \frac{9}{8}t_1 + \frac{11}{8}t_2 + \frac{13}{16}, x^4 = t_1, x^5 = t_2$

$$24. \begin{cases} -8x^1 - 7x^2 - 16x^4 - 5x^5 = -6, \\ -10x^1 - 12x^2 - 2x^3 - 18x^4 + 3x^5 = 6, \\ 4x^1 + 6x^2 + 8x^3 - 3x^5 = 12, \\ -7x^1 - 5x^2 + 3x^3 - 17x^4 - 7x^5 = -3. \end{cases}$$

Ответ: $x^1 = -2t_1 - \frac{13}{4}t_2 + 6, x^2 = 3t_2 - 6, x^3 = t_1 - 1/4t_2 + 3, x^4 = t_1, x^5 = t_2$

$$25. \begin{cases} 7x^1 + 6x^2 - 3x^3 - 6x^4 + x^5 = 1, \\ 5x^1 - 2x^2 - 5x^3 + 2x^4 + 3x^5 = 3, \\ 2x^1 + 8x^2 + 2x^3 - 8x^4 - 2x^5 = -2, \\ 7x^1 - 5x^2 - 8x^3 + 5x^4 + 5x^5 = 5. \end{cases}$$

Ответ: $x^1 = \frac{9}{11}t_1 - \frac{5}{11}t_3 + \frac{5}{11}$, $x^2 = -\frac{5}{11}t_1 + t_2 + 4/11t_3 - 4/11$, $x^3 = t_1$, $x^4 = t_2$, $x^5 = t_3$

$$26. \begin{cases} -12x^1 + 3x^2 + 3x^3 + 3x^4 = 0, \\ 3x^1 - 2x^2 - 12x^3 + 13x^4 + 15x^5 = -10, \\ 8x^1 - 3x^2 - 11x^3 + 9x^4 + 12x^5 = -8, \\ -4x^1 + 2x^2 + 10x^3 - 10x^4 - 12x^5 = 8. \end{cases}$$

Ответ: $x^1 = -2t_1 + 3t_2 + 3t_3 + 2$, $x^2 = -9t_1 + 11t_2 + 12t_3 + 8$, $x^3 = t_1$, $x^4 = t_2$, $x^5 = t_3$

$$27. \begin{cases} 7x^2 + 3x^3 - 3x^4 + 4x^5 = -1, \\ -4x^1 - 9x^2 - 9x^3 + 5x^4 - 4x^5 = 7, \\ 3x^1 - 2x^2 + 3x^3 - 2x^5 = -4, \\ -3x^1 - 12x^2 - 9x^3 + 6x^4 - 6x^5 = 6. \end{cases}$$

Ответ: $x^1 = -\frac{9}{7}t_1 + 2/7t_2 + 2/7t_3 - \frac{10}{7}$, $x^2 = -3/7t_1 + 3/7t_2 - 4/7t_3 - 1/7$, $x^3 = t_1$, $x^4 = t_2$, $x^5 = t_3$

$$28. \begin{cases} -9x^1 - 2x^2 + 7x^3 - 2x^4 - 4x^5 = -3, \\ 12x^1 - 6x^2 - 12x^3 + 2x^4 + 7x^5 = 8, \\ -11x^1 + 8x^2 + 11x^3 + 4x^4 - 6x^5 = -9, \\ 3x^1 - 8x^2 - 5x^3 + 3x^5 = 5. \end{cases}$$

Ответ: $x^1 = 6t_1 + 1/2t_2 + 1$, $x^2 = -7/3t_1 - 1/6t_2 - 2/3$, $x^3 = \frac{22}{3}t_1 + 7/6t_2 + 2/3$, $x^4 = t_1$, $x^5 = t_2$

$$29. \begin{cases} -4x^1 - 2x^2 + 2x^4 - 6x^5 = 4, \\ 5x^1 - 2x^2 - 6x^3 - 4x^4 + 3x^5 = 1, \\ -4x^1 + x^2 + 4x^3 + 3x^4 - 3x^5 = 0, \\ -5x^1 - x^2 + 2x^3 + 3x^4 - 6x^5 = 3. \end{cases}$$

Ответ: $x^1 = 2/3t_1 + 2/3t_2 - t_3 - 1/3$, $x^2 = -4/3t_1 - 1/3t_2 - t_3 - 4/3$, $x^3 = t_1$, $x^4 = t_2$, $x^5 = t_3$

$$30. \begin{cases} -x^1 + x^2 + x^3 - 3x^4 - 6x^5 = -9, \\ -3x^1 - 2x^2 + 3x^3 - 9x^4 - 3x^5 = 3, \\ 2x^1 + 3x^2 - 2x^3 + 6x^4 - 3x^5 = -12, \\ 5x^1 + 3x^2 - 5x^3 + 15x^4 + 6x^5 = -3. \end{cases}$$

Ответ: $x^1 = t_1 - 3t_2 - 3t_3 + 3$, $x^2 = 3t_3 - 6$, $x^3 = t_1$, $x^4 = t_2$, $x^5 = t_3$

Задача 1.5. Определить взаимное расположение прямой ℓ и плоскости \mathcal{P} . В случае их пересечения найти общую точку.

1. а) ℓ : $x^1 = -2 - t$, $x^2 = -2 + 4t$, $x^3 = -1 + 2t$, $x^4 = 4 - 2t$;

$$\mathcal{P} : \begin{cases} 2x^1 + 6x^2 - 11x^3 + 5 = 0, \\ -2x^1 - 6x^2 - 11x^4 + 28 = 0; \end{cases}$$

б) ℓ : $\frac{x^1 + 3}{-2} = \frac{x^2 + 1}{-2} = \frac{x^3 + 5}{-3} = \frac{x^4 - 4}{2}$;

$$\mathcal{P} : \begin{cases} x^1 = -3 + 2t_2, \\ x^2 = -3 + t_1 + 3t_2, \\ x^3 = -1 + 2t_1 - 3t_2, \\ x^4 = 2 + t_1 - t_2. \end{cases}$$

Ответ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(-1, 1, -2, 2)$

2. а) ℓ : $x^1 = -7$, $x^2 = 3 + t$, $x^3 = 1 + 2t$, $x^4 = 4 - 4t$;

$$\mathcal{P} : \begin{cases} 3x^1 + 6x^2 - 3x^3 + 6 = 0, \\ -11x^1 - 12x^2 - 3x^4 - 29 = 0; \end{cases}$$

б) ℓ : $\frac{x^1 + 1}{2} = \frac{x^2 - 3}{1} = \frac{x^3 + 3}{-3} = \frac{x^4}{3}$;

$$\mathcal{P} : \begin{cases} x^1 = -3 + 2t_1 - 2t_2, \\ x^2 = 4 + t_1 - 3t_2, \\ x^3 = 1 + 2t_1 - 3t_2, \\ x^4 = -5 + 2t_2. \end{cases}$$

Ответ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(-3, 2, 0, -3)$

3. а) ℓ : $x^1 = 3 - t$, $x^2 = -10$, $x^3 = -4 + t$, $x^4 = -6 - 2t$;

$$\mathcal{P} : \begin{cases} 2x^1 - x^2 + 2x^3 - 3 = 0, \\ -4x^1 + 5x^2 + 2x^4 + 11 = 0; \end{cases}$$

б) ℓ : $\frac{x^1 + 2}{3} = \frac{x^2 - 1}{0} = \frac{x^3 + 3}{3} = \frac{x^4 - 2}{-1}$;

$$\mathcal{P} : \begin{cases} x^1 = 6 + 2t_1 - 3t_2, \\ x^2 = 3 + 3t_1 + t_2, \\ x^3 = -3t_1 - 3t_2, \\ x^4 = -4 - 2t_1 + 3t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(1, 1, 0, 1)$

4. a) $\ell : x^1 = 3t, x^2 = 1 + 2t, x^3 = 1 + 4t, x^4 = 3 + 3t;$

$$\mathcal{P} : \begin{cases} -2x^1 + 5x^2 - x^3 - 3 = 0, \\ x^1 - x^4 + 3 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 3}{1} = \frac{x^2 + 3}{-2} = \frac{x^3 - 2}{1} = \frac{x^4 - 3}{1};$

$$\mathcal{P} : \begin{cases} x^1 = 1 + t_1, \\ x^2 = 1 + 2t_2, \\ x^3 = -3 + t_1 - 3t_2, \\ x^4 = -1 + t_1 - 2t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(2, -1, 1, 2)$

5. a) $\ell : x^1 = -6 - t, x^2 = -5 + t, x^3 = 1 - 5t, x^4 = 10;$

$$\mathcal{P} : \begin{cases} 2x^1 + 12x^2 + 2x^3 - 10 = 0, \\ 2x^4 + 6 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 3}{3} = \frac{x^2 + 6}{-3} = \frac{x^3 + 2}{-2} = \frac{x^4 - 1}{1};$

$$\mathcal{P} : \begin{cases} x^1 = -2 - 2t_1, \\ x^2 = -7 - 2t_1 + 2t_2, \\ x^3 = 2 - 2t_2, \\ x^4 = 1 + 2t_1 + t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(0, -3, 0, 0)$

6. a) $\ell : x^1 = -3 + 2t, x^2 = -1 + 2t, x^3 = -2, x^4 = 3t;$

$$\mathcal{P} : \begin{cases} -2x^1 + 2x^2 + 2x^3 = 0, \\ -x^1 - 2x^2 + 2x^4 - 5 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 4}{-3} = \frac{x^2 - 3}{-1} = \frac{x^3 + 6}{3} = \frac{x^4}{0};$

$$\mathcal{P} : \begin{cases} x^1 = -3 - t_1 + 3t_2, \\ x^2 = 7 + 3t_1 - 2t_2, \\ x^3 = -1 + 3t_1 + t_2, \\ x^4 = 2t_1 + 2t_2. \end{cases}$$

ОТВЕТ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(1, 2, -3, 0)$

7. а) $\ell : x^1 = 4 + 3t, x^2 = -7 - 4t, x^3 = 0, x^4 = 2 + 2t$;

$$\mathcal{P} : \begin{cases} 4x^1 + 3x^2 + x^3 + 5 = 0, \\ -6x^1 - 4x^2 + x^4 - 6 = 0; \end{cases}$$

б) $\ell : \frac{x^1 + 3}{1} = \frac{x^2 + 1}{1} = \frac{x^3 + 5}{3} = \frac{x^4 - 2}{0}$;

$$\mathcal{P} : \begin{cases} x^1 = -6 + 2t_1 - 2t_2, \\ x^2 = 1 + 2t_1 + 3t_2, \\ x^3 = -7 + 2t_1 - 3t_2, \\ x^4 = -t_1 - 3t_2. \end{cases}$$

ОТВЕТ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(-2, 0, -2, 2)$

8. а) $\ell : x^1 = -5 - 3t, x^2 = 2 - 4t, x^3 = 5 + 3t, x^4 = 0$;

$$\mathcal{P} : \begin{cases} -9x^1 - 9x^3 = 0, \\ -9x^4 = 0; \end{cases}$$

б) $\ell : \frac{x^1 - 2}{0} = \frac{x^2 + 3}{1} = \frac{x^3 - 3}{-1} = \frac{x^4 + 2}{3}$;

$$\mathcal{P} : \begin{cases} x^1 = -1 - t_1 - 2t_2, \\ x^2 = 2 + t_1 + 3t_2, \\ x^3 = 3 - 2t_1 + 3t_2, \\ x^4 = -t_1. \end{cases}$$

ОТВЕТ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(2, -2, 2, 1)$

9. а) $\ell : x^1 = 1 - t, x^2 = -3 + 2t, x^3 = 5 + t, x^4 = 3 + 4t$;

$$\mathcal{P} : \begin{cases} -6x^1 - 4x^2 + 2x^3 - 16 = 0, \\ -4x^1 - 6x^2 + 2x^4 - 20 = 0; \end{cases}$$

б) $\ell : \frac{x^1 - 2}{0} = \frac{x^2}{-1} = \frac{x^3}{1} = \frac{x^4 - 2}{0}$;

$$\mathcal{P} : \begin{cases} x^1 = 5 + t_1 - 2t_2, \\ x^2 = 1 - t_1 - t_2, \\ x^3 = -4 - 3t_1, \\ x^4 = 4 + 3t_1 + t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(2, 1, -1, 2)$

10. a) $\ell : x^1 = 1 - t, x^2 = 1 - 3t, x^3 = 1 - 2t, x^4 = 5 - 6t$;

$$\mathcal{P} : \begin{cases} -2x^1 - 4x^2 + 7x^3 - 1 = 0, \\ 3x^1 - 15x^2 + 7x^4 - 23 = 0; \end{cases}$$

b) $\ell : \frac{x^1}{-2} = \frac{x^2 + 5}{-2} = \frac{x^3 - 2}{-1} = \frac{x^4 + 2}{0}$;

$$\mathcal{P} : \begin{cases} x^1 = 1 + t_1, \\ x^2 = -4 + 2t_1 + t_2, \\ x^3 = 3 + 2t_1 + 2t_2, \\ x^4 = -1 - 3t_1 - 2t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(2, -3, 3, -2)$

11. a) $\ell : x^1 = -5 + 5t, x^2 = 5 + 3t, x^3 = -6 + 2t, x^4 = 1 - 2t$;

$$\mathcal{P} : \begin{cases} 5x^1 - 11x^2 + 4x^3 - 22 = 0, \\ -5x^1 + 11x^2 + 4x^4 + 22 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 4}{3} = \frac{x^2 - 1}{0} = \frac{x^3 + 2}{0} = \frac{x^4 - 3}{-1}$;

$$\mathcal{P} : \begin{cases} x^1 = -3 + 2t_1, \\ x^2 = 2 - 2t_1 - t_2, \\ x^3 = -3 - t_2, \\ x^4 = 2 + 3t_1 + 3t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(-1, 1, -2, 2)$

12. a) $\ell : x^1 = 4 - t, x^2 = 2 - 2t, x^3 = -2 + 3t, x^4 = t$;

$$\mathcal{P} : \begin{cases} 4x^1 - 8x^2 - 4x^3 - 8 = 0, \\ 6x^1 - 5x^2 - 4x^4 - 14 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 6}{3} = \frac{x^2 + 1}{2} = \frac{x^3 - 1}{-2} = \frac{x^4 - 1}{-3}$;

$$\mathcal{P} : \begin{cases} x^1 = 1 - t_1 + 3t_2, \\ x^2 = 6 - 3t_1 + 2t_2, \\ x^3 = -2 - 2t_1 - 3t_2, \\ x^4 = -1 - 2t_1 - t_2. \end{cases}$$

ОТВЕТ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(-3, 1, -1, -2)$

13. а) $\ell : x^1 = -1 - 2t, x^2 = -1 - 4t, x^3 = -2 + t, x^4 = -4 + 2t;$

$$\mathcal{P} : \begin{cases} -3x^1 - x^2 - 10x^3 - 24 = 0, \\ 6x^1 - 8x^2 - 10x^4 - 42 = 0; \end{cases}$$

б) $\ell : \frac{x^1 - 2}{3} = \frac{x^2 - 2}{0} = \frac{x^3}{-2} = \frac{x^4 - 3}{2};$

$$\mathcal{P} : \begin{cases} x^1 = -1 + 3t_1 + 3t_2, \\ x^2 = 1 - t_1 - 2t_2, \\ x^3 = 1 + 3t_1 + 2t_2, \\ x^4 = -1 - 2t_2. \end{cases}$$

ОТВЕТ: а) $\text{rank}(p, q, A - B) = 2$; б) $M(-1, 2, 2, 1)$

14. а) $\ell : x^1 = 5 + t, x^2 = -10 + 3t, x^3 = t, x^4 = -t;$

$$\mathcal{P} : \begin{cases} -2x^1 + 2x^3 - 12 = 0, \\ 5x^1 - x^2 + 2x^4 + 18 = 0; \end{cases}$$

б) $\ell : \frac{x^1 + 3}{3} = \frac{x^2 - 1}{1} = \frac{x^3 - 5}{-3} = \frac{x^4 + 2}{1};$

$$\mathcal{P} : \begin{cases} x^1 = 3t_1 - 3t_2, \\ x^2 = 2 + 2t_1 - 2t_2, \\ x^3 = -3 + 3t_1 + 2t_2, \\ x^4 = -3 + 2t_1. \end{cases}$$

ОТВЕТ: а) $\text{rank}(p, q, A - B) = 3$; б) $M(0, 2, 2, -1)$

15. а) $\ell : x^1 = 1 - t, x^2 = -1 + 2t, x^3 = 1 + t, x^4 = -1 - t;$

$$\mathcal{P} : \begin{cases} -2x^1 - 3x^2 + 4x^3 - 5 = 0, \\ -6x^1 - x^2 + 4x^4 + 9 = 0; \end{cases}$$

б) $\ell : \frac{x^1 - 3}{-2} = \frac{x^2 - 1}{-1} = \frac{x^3 - 2}{-1} = \frac{x^4 - 1}{2};$

$$\mathcal{P} : \begin{cases} x^1 = 1 + 2t_1 - 2t_2, \\ x^2 = -2t_1 + 2t_2, \\ x^3 = 1 + 2t_1 - 2t_2, \\ x^4 = 6 + 3t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(1, 0, 1, 3)$

16. a) $\ell : x^1 = -8 - t, x^2 = -1 - t, x^3 = 3 - 2t, x^4 = -5 - 5t;$

$$\mathcal{P} : \begin{cases} 3x^1 + 3x^2 - 3x^3 - 12 = 0, \\ 8x^1 + 7x^2 - 3x^4 - 11 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 4}{-1} = \frac{x^2}{1} = \frac{x^3 - 1}{-1} = \frac{x^4}{-1};$

$$\mathcal{P} : \begin{cases} x^1 = 4 - 2t_1 - t_2, \\ x^2 = 5 - 2t_1 + 2t_2, \\ x^3 = -6 + 3t_1 - 3t_2, \\ x^4 = 3 - 3t_1 + t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(3, 1, 0, -1)$

17. a) $\ell : x^1 = 2t, x^2 = -3 - t, x^3 = -5t, x^4 = 2 + t;$

$$\mathcal{P} : \begin{cases} -12x^1 + x^2 - 5x^3 + 3 = 0, \\ 3x^1 + x^2 - 5x^4 + 13 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 3}{0} = \frac{x^2 - 4}{2} = \frac{x^3 - 2}{-1} = \frac{x^4 + 2}{1};$

$$\mathcal{P} : \begin{cases} x^1 = -7 - t_1 - 3t_2, \\ x^2 = -2 - 2t_1 - 2t_2, \\ x^3 = 2 - 3t_1 + 2t_2, \\ x^4 = -7 - 2t_1 - 2t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(-3, 2, 3, -3)$

18. a) $\ell : x^1 = 6, x^2 = 5 + 2t, x^3 = 6 - 2t, x^4 = 1;$

$$\mathcal{P} : \begin{cases} -6x^1 + 4x^2 + 4x^3 + 22 = 0, \\ -4x^1 + 4x^4 + 8 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 5}{2} = \frac{x^2 + 2}{-1} = \frac{x^3 - 1}{3} = \frac{x^4 - 1}{-2};$

$$\mathcal{P} : \begin{cases} x^1 = 3t_1, \\ x^2 = 1 - 2t_1, \\ x^3 = 1 + 3t_2, \\ x^4 = -1 + 2t_1 - 2t_2. \end{cases}$$

ОТВЕТ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(3, -1, -2, 3)$

19. a) $\ell : x^1 = 2 + t, x^2 = -2 - 6t, x^3 = -2 - t, x^4 = 4$;

$$\mathcal{P} : \begin{cases} -15x^1 - 3x^2 + 3x^3 + 30 = 0, \\ 12x^1 + 2x^2 + 3x^4 - 32 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 1}{-1} = \frac{x^2 + 4}{-3} = \frac{x^3 + 2}{-1} = \frac{x^4 - 4}{3}$;

$$\mathcal{P} : \begin{cases} x^1 = 2 + t_1 - t_2, \\ x^2 = -2 + t_1, \\ x^3 = -2 + t_1, \\ x^4 = 2 - t_1. \end{cases}$$

ОТВЕТ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(2, -1, -1, 1)$

20. a) $\ell : x^1 = -8 + 4t, x^2 = -5t, x^3 = 6 - 2t, x^4 = -3 + 2t$;

$$\mathcal{P} : \begin{cases} 11x^1 + 6x^2 + 7x^3 + 40 = 0, \\ 9x^1 + 10x^2 + 7x^4 + 41 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 1}{-3} = \frac{x^2 - 1}{-2} = \frac{x^3 + 2}{1} = \frac{x^4}{0}$;

$$\mathcal{P} : \begin{cases} x^1 = 5 - t_1 + 2t_2, \\ x^2 = 1 + 2t_1, \\ x^3 = -3, \\ x^4 = 0. \end{cases}$$

ОТВЕТ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(2, 3, -3, 0)$

21. a) $\ell : x^1 = -3 + 3t, x^2 = -3 - t, x^3 = -3 + 2t, x^4 = -7$;

$$\mathcal{P} : \begin{cases} 6x^1 - 9x^3 - 9 = 0, \\ 2x^1 + 6x^2 - 9x^4 - 39 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 5}{-2} = \frac{x^2}{-3} = \frac{x^3 + 1}{-1} = \frac{x^4 - 3}{-3}$;

$$\mathcal{P} : \begin{cases} x^1 = 1 + 3t_1 - t_2, \\ x^2 = 1 - 3t_1 - t_2, \\ x^3 = -3t_1 + t_2, \\ x^4 = 1 - 2t_1 + t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(3, -3, -2, 0)$

22. a) $\ell : x^1 = 2 + 3t, x^2 = 3 + 3t, x^3 = 1 + 2t, x^4 = -1 - t$;

$$\mathcal{P} : \begin{cases} -2x^2 + 3x^3 + 3 = 0, \\ 6x^1 - 5x^2 + 3x^4 + 6 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 1}{-1} = \frac{x^2 + 1}{2} = \frac{x^3 - 1}{3} = \frac{x^4 - 6}{3}$;

$$\mathcal{P} : \begin{cases} x^1 = 8 - 3t_1 - 3t_2, \\ x^2 = -4 + 2t_1 - t_2, \\ x^3 = -5 + 2t_1 + t_2, \\ x^4 = 6 - t_1 - 2t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(2, -3, -2, 3)$

23. a) $\ell : x^1 = -4 + t, x^2 = 5 + 4t, x^3 = -3t, x^4 = -3 + 5t$;

$$\mathcal{P} : \begin{cases} -6x^1 + 3x^2 + 2x^3 - 23 = 0, \\ -2x^1 - 2x^2 + 2x^4 - 4 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 5}{-3} = \frac{x^2 + 1}{0} = \frac{x^3}{-2} = \frac{x^4 + 3}{3}$;

$$\mathcal{P} : \begin{cases} x^1 = -1 - 2t_1 - t_2, \\ x^2 = -5 - 3t_1 - t_2, \\ x^3 = -1 + t_1, \\ x^4 = 1 + t_1. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(2, -1, -2, 0)$

24. a) $\ell : x^1 = -4 + 5t, x^2 = 1 + 2t, x^3 = -4 + t, x^4 = 4 - 3t$;

$$\mathcal{P} : \begin{cases} -3x^2 + 6x^3 + 27 = 0, \\ 6x^1 - 6x^2 + 6x^4 + 6 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 2}{1} = \frac{x^2 - 1}{1} = \frac{x^3 - 1}{0} = \frac{x^4 - 2}{-3}$;

$$\mathcal{P} : \begin{cases} x^1 = 3 - 3t_1 - 3t_2, \\ x^2 = 3 + t_2, \\ x^3 = 2 - t_1, \\ x^4 = -t_1. \end{cases}$$

ОТВЕТ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(3, 2, 1, -1)$

25. a) $\ell : x^1 = -3 + 4t, x^2 = 8, x^3 = -1 - t, x^4 = -1 + t;$

$$\mathcal{P} : \begin{cases} 3x^1 + 7x^2 + 12x^3 - 35 = 0, \\ -3x^1 - 3x^2 + 12x^4 + 27 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 1}{1} = \frac{x^2 - 1}{1} = \frac{x^3}{-1} = \frac{x^4 + 5}{-3};$

$$\mathcal{P} : \begin{cases} x^1 = -1 + t_1, \\ x^2 = 2 - 3t_1 - t_2, \\ x^3 = 1 - t_1 - t_2, \\ x^4 = 3 - 2t_1 + 3t_2. \end{cases}$$

ОТВЕТ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(0, 0, 1, -2)$

26. a) $\ell : x^1 = -7 + 2t, x^2 = -4 - t, x^3 = -2 + t, x^4 = 1 + t;$

$$\mathcal{P} : \begin{cases} x^1 + 7x^2 + 5x^3 - 17 = 0, \\ -4x^1 - 3x^2 + 5x^4 + 13 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 1}{-3} = \frac{x^2 + 2}{1} = \frac{x^3 + 4}{-2} = \frac{x^4 + 2}{-2};$

$$\mathcal{P} : \begin{cases} x^1 = 1 - 2t_1 + 3t_2, \\ x^2 = -6 + t_1 + 2t_2, \\ x^3 = -1 - 3t_1 + 2t_2, \\ x^4 = 3t_1 - 3t_2. \end{cases}$$

ОТВЕТ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(2, -3, -2, 0)$

27. a) $\ell : x^1 = 2 + 3t, x^2 = -9, x^3 = 1 + 3t, x^4 = -5 + 2t;$

$$\mathcal{P} : \begin{cases} 9x^1 + 3x^2 - 9x^3 + 18 = 0, \\ 6x^1 + 9x^2 - 9x^4 + 24 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 3}{-3} = \frac{x^2 + 2}{1} = \frac{x^3 - 5}{3} = \frac{x^4 - 1}{-2};$

$$\mathcal{P} : \begin{cases} x^1 = -1 + t_1 - 2t_2, \\ x^2 = 2 + 2t_1 + 3t_2, \\ x^3 = 3 + t_1, \\ x^4 = 3 + t_1 - t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(0, -3, 2, 3)$

28. a) $\ell : x^1 = 3t, x^2 = 3 + t, x^3 = -1 - 5t, x^4 = 4;$

$$\mathcal{P} : \begin{cases} -8x^1 + 9x^2 - 3x^3 - 8 = 0, \\ 3x^1 - 9x^2 - 3x^4 + 24 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 3}{-2} = \frac{x^2 - 1}{2} = \frac{x^3 + 2}{1} = \frac{x^4 + 2}{0};$

$$\mathcal{P} : \begin{cases} x^1 = 7 - 3t_1 + 3t_2, \\ x^2 = 6 - t_1 + 2t_2, \\ x^3 = -2t_1 - t_2, \\ x^4 = -6 + 2t_1 - 2t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 3$; b) $M(1, 3, -1, -2)$

29. a) $\ell : x^1 = -6 - 6t, x^2 = 2 + 4t, x^3 = -2 - 4t, x^4 = -2 - t;$

$$\mathcal{P} : \begin{cases} -6x^2 - 6x^3 = 0, \\ -x^1 - 3x^2 - 6x^4 - 12 = 0; \end{cases}$$

b) $\ell : \frac{x^1 + 2}{3} = \frac{x^2 + 2}{1} = \frac{x^3}{-3} = \frac{x^4 + 3}{2};$

$$\mathcal{P} : \begin{cases} x^1 = 2 + t_1, \\ x^2 = -2 + 2t_1 - 3t_2, \\ x^3 = 1 + t_1 + 3t_2, \\ x^4 = -1 + t_1 - t_2. \end{cases}$$

Ответ: a) $\text{rank}(p, q, A - B) = 2$; b) $M(1, -1, -3, -1)$

30. a) $\ell : x^1 = 8 - 3t, x^2 = 3 + 3t, x^3 = -8 + 3t, x^4 = -6;$

$$\mathcal{P} : \begin{cases} 9x^1 + 9x^3 + 9 = 0, \\ -6x^1 - 6x^2 + 9x^4 - 18 = 0; \end{cases}$$

b) $\ell : \frac{x^1 - 3}{0} = \frac{x^2 - 1}{2} = \frac{x^3 - 4}{-2} = \frac{x^4 - 4}{-1};$

$$\mathcal{P} : \begin{cases} x^1 = 1 - 2t_1, \\ x^2 = 2 - 2t_1 - t_2, \\ x^3 = -3 - 2t_1 + 3t_2, \\ x^4 = 4 + t_1. \end{cases}$$

Ответ: а) $\text{rank}(p, q, A - B) = 3$; б) $M(3, 3, 2, 3)$

Задача 1.6. Определить взаимное расположение плоскостей $\mathcal{P}_1 = A + \langle \vec{u}, \vec{v} \rangle$ и $\mathcal{P}_2 = B + \langle \vec{p}, \vec{q} \rangle$.

1. $A(0, 2, 2, 0, -2)$, $\vec{u} = (-1, 3, 4, 5, -4)$, $\vec{v} = (-2, 5, -5, 5, -1)$;
 $B(0, -1, 1, -2, 0)$, $\vec{p} = (5, -4, -3, -4, -3)$, $\vec{q} = (5, -3, 1, 5, -2)$.

Ответ: $(4, 5)$.

2. $A(2, 2, -1, 2)$, $\vec{u} = (1, -1, -4, 8)$, $\vec{v} = (5, 3, 4, -8)$;
 $B(-1, -1, 1, 1)$, $\vec{p} = (19, -16, -9, 12)$, $\vec{q} = (-5, 20, 7, -8)$.

Ответ: $(3, 4)$, 123 , $\vec{v} = (7, 2, -1, 2)$.

3. $A(-1, 2, 0, -2)$, $\vec{u} = (3, 0, -3, -3)$, $\vec{v} = (-1, 4, -5, -1)$;
 $B(-1, -2, -1, 1)$, $\vec{p} = (10, -10, 4, 2)$, $\vec{q} = (18, -14, 0, 10)$.

Ответ: $(3, 4)$, 123 , $\vec{v} = (6, -8, 6, -2)$.

4. $A(-2, 0, -2, 1, 0)$, $\vec{u} = (-5, -1, -2, -4, -5)$, $\vec{v} = (5, 1, 1, -3, 2)$;
 $B(-2, 0, 1, -2, 0)$, $\vec{p} = (-1, -4, 1, 4, 5)$, $\vec{q} = (1, 1, 1, -2, 1)$.

Ответ: $(4, 5)$.

5. $A(-2, 2, -1, -2, 0)$, $\vec{u} = (2, 2, 4, 3, -4)$, $\vec{v} = (-6, -6, 10, 2, -10)$;
 $B(-2, 0, 2, 1, 0)$, $\vec{p} = (5, 9, 10, 7, 8)$, $\vec{q} = (7, 3, -10, -1, -8)$.

Ответ: $(3, 4)$, 123 , $\vec{v} = (6, 6, 0, 3, 0)$.

6. $A(3, -5, 4, 2, -3)$, $\vec{u} = (1, -2, 2, 0, 0)$, $\vec{v} = (-1, 3, -2, 0, 1)$;
 $B(1, 0, 0, 2, -2)$, $\vec{p} = (3, 2, -1, -5, 8)$, $\vec{q} = (3, 2, -1, -5, 8)$.

Ответ: $(3, 3)$, M , $(1, 0, 0, 2, -2)$, 123 , $\vec{v} = (1, 1, 2, 0, 3)$.

7. $A(1, 0, 0, 1)$, $\vec{u} = (3, -1, -1, 2)$, $\vec{v} = (5, -2, -2, 2)$;
 $B(9, -33, -28, -25)$, $\vec{p} = (5, -20, -17, -16)$, $\vec{q} = (5, -14, -12, -10)$.

Ответ: $(3, 3)$, M , $(-1, 1, 1, 1)$, 123 , $\vec{v} = (5, -2, -2, 2)$.

8. $A(0, -2, 0, 2, -1)$, $\vec{u} = (-2, -6, 0, -8, -8)$, $\vec{v} = (-1, 4, 0, 10, 10)$;
 $B(0, -2, -1, -2, 2)$, $\vec{p} = (-6, 0, 5, -9, -9)$, $\vec{q} = (22, -8, -15, 3, 3)$.

Ответ: $(3, 4)$, 123, $\vec{v} = (1, -2, 0, -6, -6)$.

9. $A(1, 0, -1, -1)$, $\vec{u} = (-5, 4, -5, 1)$, $\vec{v} = (-4, -1, -4, -2)$;
 $B(-2, 0, -2, 2)$, $\vec{p} = (1, 4, 1, 3)$, $\vec{q} = (0, 3, 0, 2)$.

Ответ: $(2, 3)$.

10. $A(3, 7, -5, 0, 9)$, $\vec{u} = (1, 3, -5, -1, 4)$, $\vec{v} = (-1, -2, 1, -1, -3)$;
 $B(4, 10, -2, 7, 9)$, $\vec{p} = (-1, -3, 1, -4, -4)$, $\vec{q} = (2, 5, -2, 3, 3)$.

Ответ: $(4, 4)$, 123, M , $(1, 2, 1, 0, 2)$.

11. $A(2, 2, 1, 0, 1)$, $\vec{u} = (-4, 0, -5, 1, -2)$, $\vec{v} = (-5, 0, -1, -4, 1)$;
 $B(-2, 2, -1, 1, 1)$, $\vec{p} = (4, -8, -7, -10, -3)$, $\vec{q} = (18, -24, -15, -30, -7)$.

Ответ: $(3, 4)$, 123, $\vec{v} = (-3, 0, -3, 0, -1)$.

12. $A(-1, -7, 5, 5)$, $\vec{u} = (-2, 2, 2, -8)$, $\vec{v} = (5, 4, -5, 2)$;
 $B(-6, -19, -10, 19)$, $\vec{p} = (16, 26, 14, -20)$, $\vec{q} = (8, 8, 2, 0)$.

Ответ: $(3, 3)$, M , $(2, -1, 2, -1)$, 123, $\vec{v} = (4, -1, -4, 10)$.

13. $A(4, 7, -12, 3, 7)$, $\vec{u} = (4, 1, -5, -1, 3)$, $\vec{v} = (-1, -4, 5, -4, -5)$;
 $B(2, 8, -2, 1, -5)$, $\vec{p} = (1, -2, -1, 3, -1)$, $\vec{q} = (-4, -4, 1, -4, 5)$.

Ответ: $(4, 4)$, 123, M , $(-1, 2, -2, 0, -1)$.

14. $A(2, -2, -2, -2)$, $\vec{u} = (0, 0, -10, -7)$, $\vec{v} = (0, 0, -4, -3)$;
 $B(-1, 1, -2, -2)$, $\vec{p} = (-4, 12, -20, -5)$, $\vec{q} = (-6, 18, -29, -8)$.

Ответ: $(3, 4)$, 123, $\vec{v} = (0, 0, -2, 1)$.

15. $A(-1, -2, -2, 0, -1)$, $\vec{u} = (-7, 6, 7, -2, 1)$, $\vec{v} = (3, 6, -3, 3, 6)$;
 $B(-1, 1, 0, 1, 0)$, $\vec{p} = (-4, -4, 4, -3, -5)$, $\vec{q} = (3, -2, -3, 1, 0)$.

Ответ: $(2, 3)$.

16. $A(-1, -3, 0, 0)$, $\vec{u} = (1, -5, 6, 5)$, $\vec{v} = (5, -1, -2, 1)$;
 $B(3, -5, 0, 2)$, $\vec{p} = (-6, 6, -4, -6)$, $\vec{q} = (-1, -1, 2, 1)$.

Ответ: $(2, 2)$.

17. $A(-1, -4, 0, 2)$, $\vec{u} = (0, -4, 4, 2)$, $\vec{v} = (0, -2, 4, 0)$;
 $B(-10, -8, -2, -3)$, $\vec{p} = (-27, -16, -10, -9)$, $\vec{q} = (-18, -10, -8, -6)$.

Ответ: $(3, 3)$, M , $(-1, -2, 0, 0)$, 123, $\vec{v} = (0, 2, -4, 0)$.

18. $A(2, 1, 1, 3)$, $\vec{u} = (0, 9, -3, -9)$, $\vec{v} = (2, -5, 5, 7)$;
 $B(-2, -1, -5, 1)$, $\vec{p} = (1, 8, -1, -7)$, $\vec{q} = (0, -9, 3, 9)$.

Ответ: (2, 2).

19. $A(0, -1, 0, -2, -2)$, $\vec{u} = (-1, -2, 0, -2, -3)$, $\vec{v} = (-9, -6, 0, 6, -3)$;
 $B(2, 1, 0, -2, 0)$, $\vec{p} = (0, -1, 0, -2, -2)$, $\vec{q} = (-3, 0, 0, 6, 3)$.

Ответ: (2, 2).

20. $A(1, 1, 1, -2)$, $\vec{u} = (2, 2, 0, -2)$, $\vec{v} = (1, 8, 0, -8)$;
 $B(1, -1, -1, 0)$, $\vec{p} = (1, -2, 0, 2)$, $\vec{q} = (-1, 0, 0, 0)$.

Ответ: (2, 3).

21. $A(-3, 0, 2, -2, -4)$, $\vec{u} = (8, 2, 0, 8, 4)$, $\vec{v} = (-6, 0, 0, -6, -4)$;
 $B(-1, -4, 2, 0, 0)$, $\vec{p} = (-7, 2, 0, -7, -6)$, $\vec{q} = (0, 6, 0, 0, -4)$.

Ответ: (2, 2).

22. $A(8, -2, 1, 2)$, $\vec{u} = (3, 2, 3, -1)$, $\vec{v} = (5, -3, -1, 1)$;
 $B(-7, -7, 0, -1)$, $\vec{p} = (2, 3, -3, -1)$, $\vec{q} = (5, 3, 2, 4)$.

Ответ: (4, 4), 123, M , (0, -1, -1, 2).

23. $A(-6, -1, 7, 6)$, $\vec{u} = (-4, 1, 3, 4)$, $\vec{v} = (-4, -3, 3, 4)$;
 $B(7, -8, -7, 3)$, $\vec{p} = (1, -5, -3, 4)$, $\vec{q} = (-4, 4, 5, -1)$.

Ответ: (4, 4), 123, M , (2, 1, 1, -2).

24. $A(10, 8, -9, -7, -5)$, $\vec{u} = (5, 2, -2, -1, -5)$, $\vec{v} = (-5, -5, 5, 4, -1)$;
 $B(0, 0, 4, 1, 5)$, $\vec{p} = (3, -2, -5, 2, -2)$, $\vec{q} = (-3, 3, -1, -5, -4)$.

Ответ: (4, 4), 123, M , (0, 1, -2, -2, -1).

25. $A(2, 0, -4, -9)$, $\vec{u} = (-2, -3, -5, -4)$, $\vec{v} = (-5, -5, -2, 4)$;
 $B(-8, -8, 0, -1)$, $\vec{p} = (3, 3, -5, 5)$, $\vec{q} = (-4, -3, -4, 5)$.

Ответ: (4, 4), 123, M , (-1, -2, -1, -1).

26. $A(0, 1, 2, 2, -1)$, $\vec{u} = (0, 2, 2, -4, 3)$, $\vec{v} = (8, -10, 6, -4, 1)$;
 $B(-1, -1, -2, 1, 2)$, $\vec{p} = (-1, -2, -4, 7, -5)$, $\vec{q} = (-4, 2, -6, 8, -5)$.

Ответ: (2, 3).

27. $A(6, -2, 0, -5, 2)$, $\vec{u} = (-2, -7, 3, 2, -4)$, $\vec{v} = (-2, -4, 2, 2, -3)$;
 $B(2, 2, 0, -1, 0)$, $\vec{p} = (12, -3, -3, -12, 9)$, $\vec{q} = (8, 1, -3, -8, 7)$.

Ответ: (2, 2).

28. $A(0, -2, -2, -1)$, $\vec{u} = (-8, 0, 4, 2)$, $\vec{v} = (6, 1, -2, -1)$;
 $B(-2, 2, -2, 2)$, $\vec{p} = (10, 1, -4, -2)$, $\vec{q} = (8, -2, -6, -3)$.
Ответ: $(2, 3)$.
29. $A(-2, 1, 4, 0, -1)$, $\vec{u} = (-4, 6, -5, -5, 0)$, $\vec{v} = (-8, 6, -1, -7, 0)$;
 $B(12, -15, -4, 16, 5)$, $\vec{p} = (-25, 30, 7, -29, -9)$, $\vec{q} = (-15, 14, 3, -15, -3)$.
Ответ: $(3, 3)$, $M, (2, 1, 0, 2, -1)$, 123 , $\vec{v} = (-10, 6, 1, -8, 0)$.
30. $A(-1, -2, -1, -1, 1)$, $\vec{u} = (4, 0, -4, 0, -2)$, $\vec{v} = (6, -1, -6, 0, -3)$;
 $B(0, 2, -1, -1, -1)$, $\vec{p} = (4, 0, -4, 0, -2)$, $\vec{q} = (4, -4, -4, 0, -2)$.
Ответ: $(2, 3)$.

Задача 1.7. Написать формулы перехода к новой системе координат на прямой, если за ее начало принята точка O' , а за базисный вектор — вектор $\overrightarrow{O'A}$. Координаты точек указаны в старой системе координат.

- | | | |
|--|--|---|
| 1. $O'(-4)$, $A(-2)$.
Ответ: $x' = 1/2 x + 2$ | Ответ: $x' = 1/2 x - 5$ | 12. $O'(5)$, $A(6)$.
Ответ: $x' = x - 5$ |
| 2. $O'(-9)$, $A(-7)$.
Ответ: $x' = 1/2 x + 9/2$ | 7. $O'(5)$, $A(8)$.
Ответ: $x' = 1/3 x - 5/3$ | 13. $O'(-1)$, $A(-3)$.
Ответ: $x' = -1/2 x - 1/2$ |
| 3. $O'(7)$, $A(6)$.
Ответ: $x' = -x + 7$ | 8. $O'(-8)$, $A(-9)$.
Ответ: $x' = -x - 8$ | 14. $O'(-6)$, $A(-3)$.
Ответ: $x' = 1/3 x + 2$ |
| 4. $O'(-1)$, $A(-4)$.
Ответ: $x' = -1/3 x - 1/3$ | 9. $O'(-10)$, $A(-8)$.
Ответ: $x' = 1/2 x + 5$ | 15. $O'(-7)$, $A(-9)$.
Ответ: $x' = -1/2 x - 7/2$ |
| 5. $O'(9)$, $A(12)$.
Ответ: $x' = 1/3 x - 3$ | 10. $O'(10)$, $A(8)$.
Ответ: $x' = -1/2 x + 5$ | 16. $O'(6)$, $A(5)$.
Ответ: $x' = -x + 6$ |
| 6. $O'(10)$, $A(12)$.
Ответ: $x' = 1/3 x - 3$ | 11. $O'(-8)$, $A(-10)$.
Ответ: $x' = -1/2 x - 4$ | 17. $O'(10)$, $A(13)$.
Ответ: $x' = 1/3 x - 10/3$ |

18. $O'(-3), A(-6)$. **Ответ:** $x' = -x + 8$
Ответ: $x' = -1/3 x - 1$
19. $O'(1), A(3)$. **Ответ:** $x' = -x - 6$
Ответ: $x' = 1/2 x - 1/2$
20. $O'(-7), A(-10)$. **Ответ:** $x' = 1/3 x - 7/3$
Ответ: $x' = -1/3 x - 7/3$
21. $O'(2), A(0)$. **Ответ:** $x' = 1/2 x - 5/2$
Ответ: $x' = -1/2 x + 1$
22. $O'(8), A(7)$. **Ответ:** $x' = 1/2 x - 3$
23. $O'(-6), A(-7)$. **Ответ:** $x' = -1/2 x + 3$
24. $O'(7), A(10)$. **Ответ:** $x' = 1/3 x - 1/3$
25. $O'(5), A(7)$. **Ответ:** $x' = 1/3 x + 8/3$
26. $O'(6), A(8)$. **Ответ:** $x' = 1/2 x - 7/2$
27. $O'(6), A(4)$. **Ответ:** $x' = -1/2 x + 3$
28. $O'(1), A(4)$. **Ответ:** $x' = 1/3 x - 1/3$
29. $O'(-8), A(-5)$. **Ответ:** $x' = 1/3 x + 8/3$
30. $O'(7), A(9)$. **Ответ:** $x' = 1/2 x - 7/2$

Задача 1.8. Пусть точки A, B, C, D — последовательные вершины параллелограмма, E — точка пересечения его диагоналей. Найти формулы перехода от старой системы координат $O \vec{e}_1 \vec{e}_2$ к новой $O' \vec{e}'_1 \vec{e}'_2$.

1. $O = E, \vec{e}_1 = \overrightarrow{CE}, \vec{e}_2 = \overrightarrow{EB}; O' = C, \vec{e}'_1 = \overrightarrow{CB}, \vec{e}'_2 = \overrightarrow{AB}$.

Ответ: $\{y = x' + y', x = -1 + x' - y'\},$
 $\{y' = -1/2 + 1/2 y - 1/2 x, x' = 1/2 y + 1/2 + 1/2 x\}$

2. $O = B, \vec{e}_1 = \overrightarrow{DC}, \vec{e}_2 = \overrightarrow{EA}; O' = A, \vec{e}'_1 = \overrightarrow{BE}, \vec{e}'_2 = \overrightarrow{AD}$.

Ответ: $\{x = -1 - x' - y', y = -x' - 2y'\},$
 $\{x' = -2x - 2 + y, y' = x + 1 - y\}$

3. $O = E, \vec{e}_1 = \overrightarrow{CA}, \vec{e}_2 = \overrightarrow{BE}; O' = C, \vec{e}'_1 = \overrightarrow{DA}, \vec{e}'_2 = \overrightarrow{CD}$.

Ответ: $\{y = -x' + y', x = -1/2 + 1/2 x' + 1/2 y'\},$
 $\{y' = x + 1/2 y + 1/2, x' = -1/2 y + x + 1/2\}$

4. $O = C, \vec{e}_1 = \overrightarrow{AE}, \vec{e}_2 = \overrightarrow{BC}; O' = A, \vec{e}'_1 = \overrightarrow{BA}, \vec{e}'_2 = \overrightarrow{DB}$.

Ответ: $\{y = x' - 2y', x = -2 - 2x' + 2y'\},$
 $\{y' = -y - 1 - 1/2 x, x' = -2 - x - y\}$

$$5. O = D, \vec{e}_1 = \overrightarrow{CA}, \vec{e}_2 = \overrightarrow{AD}; \quad O' = B, \vec{e}_1' = \overrightarrow{AB}, \vec{e}_2' = \overrightarrow{EB}.$$

$$\text{ОТВЕТ: } \{y = -2 - x' - y', x = -1 - x' - 1/2 y'\}, \\ \{x' = -2x + y, y' = -2 - 2y + 2x\}$$

$$6. O = E, \vec{e}_1 = \overrightarrow{BA}, \vec{e}_2 = \overrightarrow{CB}; \quad O' = B, \vec{e}_1' = \overrightarrow{CA}, \vec{e}_2' = \overrightarrow{EB}.$$

$$\text{ОТВЕТ: } \{y = x' + 1/2 + 1/2 y', x = x' - 1/2 - 1/2 y'\}, \\ \{y' = -1 + y - x, x' = 1/2 x + 1/2 y\}$$

$$7. O = C, \vec{e}_1 = \overrightarrow{AE}, \vec{e}_2 = \overrightarrow{EB}; \quad O' = B, \vec{e}_1' = \overrightarrow{AB}, \vec{e}_2' = \overrightarrow{DA}.$$

$$\text{ОТВЕТ: } \{x = -1 + x' - y', y = 1 + x' + y'\}, \\ \{y' = -1 + 1/2 y - 1/2 x, x' = 1/2 x + 1/2 y\}$$

$$8. O = C, \vec{e}_1 = \overrightarrow{CB}, \vec{e}_2 = \overrightarrow{CD}; \quad O' = D, \vec{e}_1' = \overrightarrow{EA}, \vec{e}_2' = \overrightarrow{DB}.$$

$$\text{ОТВЕТ: } \{y = 1 - y' + 1/2 x', x = y' + 1/2 x'\}, \\ \{y' = 1/2 - 1/2 y + 1/2 x, x' = -1 + x + y\}$$

$$9. O = E, \vec{e}_1 = \overrightarrow{EA}, \vec{e}_2 = \overrightarrow{CD}; \quad O' = D, \vec{e}_1' = \overrightarrow{AD}, \vec{e}_2' = \overrightarrow{BE}.$$

$$\text{ОТВЕТ: } \{x = -1 - 2x' - y', y = 1 + x' + y'\}, \\ \{x' = -x - y, y' = 2y - 1 + x\}$$

$$10. O = B, \vec{e}_1 = \overrightarrow{EC}, \vec{e}_2 = \overrightarrow{BC}; \quad O' = D, \vec{e}_1' = \overrightarrow{BA}, \vec{e}_2' = \overrightarrow{DB}.$$

$$\text{ОТВЕТ: } \{y = 2 + x' - 2y', x = -2 - 2x' + 2y'\}, \\ \{x' = -x - y, y' = 1 - y - 1/2 x\}$$

$$11. O = C, \vec{e}_1 = \overrightarrow{EB}, \vec{e}_2 = \overrightarrow{CD}; \quad O' = E, \vec{e}_1' = \overrightarrow{CA}, \vec{e}_2' = \overrightarrow{DA}.$$

$$\text{ОТВЕТ: } \{y = 1 + 2x' + y', x = 1 + 2x' + 2y'\}, \\ \{y' = x - y, x' = -1/2 + y - 1/2 x\}$$

$$12. O = B, \vec{e}_1 = \overrightarrow{DC}, \vec{e}_2 = \overrightarrow{ED}; \quad O' = A, \vec{e}_1' = \overrightarrow{AE}, \vec{e}_2' = \overrightarrow{AD}.$$

$$\text{ОТВЕТ: } \{y = x' + 2y', x = -1 + x' + y'\}, \\ \{y' = -1 + y - x, x' = 2x + 2 - y\}$$

$$13. O = C, \vec{e}_1 = \overrightarrow{BA}, \vec{e}_2 = \overrightarrow{CB}; \quad O' = A, \vec{e}_1' = \overrightarrow{BD}, \vec{e}_2' = \overrightarrow{EC}.$$

$$\text{ОТВЕТ: } \{y = 1 - x' - 1/2 y', x = 1 + x' - 1/2 y'\}, \\ \{y' = 2 - x - y, x' = -1/2 y + 1/2 x\}$$

$$14. O = A, \vec{e}_1 = \overrightarrow{BE}, \vec{e}_2 = \overrightarrow{AD}; \quad O' = E, \vec{e}_1' = \overrightarrow{AE}, \vec{e}_2' = \overrightarrow{DC}.$$

$$\text{ОТВЕТ: } \{y = 1 + x' + y', x = -1 - x' - 2y'\}, \\ \{y' = -x - y, x' = 2y - 1 + x\}$$

$$15. O = B, \vec{e}_1 = \overrightarrow{DA}, \vec{e}_2 = \overrightarrow{BA}; \quad O' = E, \vec{e}'_1 = \overrightarrow{DB}, \vec{e}'_2 = \overrightarrow{CE}.$$

$$\text{ОТВЕТ: } \{y = -x' + 1/2 + 1/2 y', x = x' - 1/2 + 1/2 y'\}, \\ \{y' = x + y, x' = 1/2 - 1/2 y + 1/2 x\}$$

$$16. O = E, \vec{e}_1 = \overrightarrow{AB}, \vec{e}_2 = \overrightarrow{AD}; \quad O' = C, \vec{e}'_1 = \overrightarrow{CE}, \vec{e}'_2 = \overrightarrow{DE}.$$

$$\text{ОТВЕТ: } \{y = 1/2 - 1/2 x' - 1/2 y', x = 1/2 - 1/2 x' + 1/2 y'\}, \\ \{x' = 1 - x - y, y' = x - y\}$$

$$17. O = D, \vec{e}_1 = \overrightarrow{DC}, \vec{e}_2 = \overrightarrow{CA}; \quad O' = B, \vec{e}'_1 = \overrightarrow{BC}, \vec{e}'_2 = \overrightarrow{BD}.$$

$$\text{ОТВЕТ: } \{y = 1 - x' - y', x = 2 - x' - 2y'\}, \\ \{x' = -2y + x, y' = -x + y + 1\}$$

$$18. O = B, \vec{e}_1 = \overrightarrow{BA}, \vec{e}_2 = \overrightarrow{BC}; \quad O' = A, \vec{e}'_1 = \overrightarrow{CE}, \vec{e}'_2 = \overrightarrow{BE}.$$

$$\text{ОТВЕТ: } \{y = -1/2 x' + 1/2 y', x = 1 + 1/2 x' + 1/2 y'\}, \\ \{x' = -1 - y + x, y' = -1 + x + y\}$$

$$19. O = D, \vec{e}_1 = \overrightarrow{AD}, \vec{e}_2 = \overrightarrow{DC}; \quad O' = A, \vec{e}'_1 = \overrightarrow{BD}, \vec{e}'_2 = \overrightarrow{EC}.$$

$$\text{ОТВЕТ: } \{y = -x' + 1/2 y', x = -1 + x' + 1/2 y'\}, \\ \{x' = 1/2 - 1/2 y + 1/2 x, y' = x + y + 1\}$$

$$20. O = C, \vec{e}_1 = \overrightarrow{AE}, \vec{e}_2 = \overrightarrow{DC}; \quad O' = E, \vec{e}'_1 = \overrightarrow{DE}, \vec{e}'_2 = \overrightarrow{CB}.$$

$$\text{ОТВЕТ: } \{y = x' + y', x = -1 - x' - 2y'\}, \\ \{x' = 2y + x + 1, y' = -x - y - 1\}$$

$$21. O = C, \vec{e}_1 = \overrightarrow{BA}, \vec{e}_2 = \overrightarrow{DE}; \quad O' = B, \vec{e}'_1 = \overrightarrow{DA}, \vec{e}'_2 = \overrightarrow{EA}.$$

$$\text{ОТВЕТ: } \{y = 2 + 2x' + y', x = 1 + x' + y'\}, \\ \{x' = -1 + y - x, y' = 2x - y\}$$

$$22. O = B, \vec{e}_1 = \overrightarrow{CE}, \vec{e}_2 = \overrightarrow{BC}; \quad O' = D, \vec{e}'_1 = \overrightarrow{ED}, \vec{e}'_2 = \overrightarrow{BA}.$$

$$\text{ОТВЕТ: } \{x = 2 + x' + 2y', y = 2 + x' + y'\}, \\ \{x' = -2 + 2y - x, y' = x - y\}$$

$$23. O = B, \vec{e}_1 = \overrightarrow{EA}, \vec{e}_2 = \overrightarrow{AB}; \quad O' = A, \vec{e}'_1 = \overrightarrow{BD}, \vec{e}'_2 = \overrightarrow{DA}.$$

$$\text{ОТВЕТ: } \{x = -2x' + 2y', y = -1 - 2x' + y'\}, \\ \{y' = -1 - y + x, x' = -1 - y + 1/2 x\}$$

$$24. O = D, \vec{e}_1 = \overrightarrow{AB}, \vec{e}_2 = \overrightarrow{AD}; \quad O' = A, \vec{e}'_1 = \overrightarrow{EC}, \vec{e}'_2 = \overrightarrow{DB}.$$

$$\text{ОТВЕТ: } \{y = -1 + 1/2 x' - y', x = y' + 1/2 x'\}, \\ \{y' = -1/2 - 1/2 y + 1/2 x, x' = x + y + 1\}$$

$$25. O = C, \vec{e}_1 = \overrightarrow{EB}, \vec{e}_2 = \overrightarrow{EA}; \quad O' = E, \vec{e}'_1 = \overrightarrow{CD}, \vec{e}'_2 = \overrightarrow{BC}.$$

$$\text{Ответ: } \{y = 1 + x' - y', x = -x' - y'\}, \\ \{y' = 1/2 - 1/2x - 1/2y, x' = -1/2 + 1/2y - 1/2x\}$$

$$26. O = A, \vec{e}_1 = \overrightarrow{DE}, \vec{e}_2 = \overrightarrow{AD}; \quad O' = D, \vec{e}'_1 = \overrightarrow{AC}, \vec{e}'_2 = \overrightarrow{DC}.$$

$$\text{Ответ: } \{y = 1 + 2x' + y', x = 2x' + 2y'\}, \\ \{y' = 1 - y + x, x' = -1 + y - 1/2x\}$$

$$27. O = C, \vec{e}_1 = \overrightarrow{BC}, \vec{e}_2 = \overrightarrow{DB}; \quad O' = D, \vec{e}'_1 = \overrightarrow{BA}, \vec{e}'_2 = \overrightarrow{AE}.$$

$$\text{Ответ: } \{y = -1 - x' + 1/2y', x = -1 - x' + y'\}, \\ \{x' = x - 2y - 1, y' = 2x - 2y\}$$

$$28. O = C, \vec{e}_1 = \overrightarrow{CD}, \vec{e}_2 = \overrightarrow{DE}; \quad O' = A, \vec{e}'_1 = \overrightarrow{CB}, \vec{e}'_2 = \overrightarrow{AE}.$$

$$\text{Ответ: } \{y = 2 + 2x' - y', x = 2 + x' - y'\}, \\ \{x' = -x + y, y' = 2 - 2x + y\}$$

$$29. O = B, \vec{e}_1 = \overrightarrow{CA}, \vec{e}_2 = \overrightarrow{DE}; \quad O' = E, \vec{e}'_1 = \overrightarrow{BC}, \vec{e}'_2 = \overrightarrow{AB}.$$

$$\text{Ответ: } \{y = -1 - x' + y', x = -1/2x' - 1/2y'\}, \\ \{y' = 1/2 + 1/2y - x, x' = -x - 1/2 - 1/2y\}$$

$$30. O = E, \vec{e}_1 = \overrightarrow{BE}, \vec{e}_2 = \overrightarrow{DC}; \quad O' = B, \vec{e}'_1 = \overrightarrow{CA}, \vec{e}'_2 = \overrightarrow{CB}.$$

$$\text{Ответ: } \{x = -2x' - 2y' - 1, y = -2x' - y'\}, \\ \{y' = -1 + y - x, x' = 1/2 - y + 1/2x\}$$

Задача 1.9. На плоскости заданы две системы координат: старая Oxy и новая $O'x'y'$, связанные указанными соотношениями, и две прямые ℓ_1 и ℓ_2 , заданные своими уравнениями в старой и новой системах координат соответственно. Найти:

- i. Координаты точки O' и единичных векторов \vec{e}'_1, \vec{e}'_2 осей $O'x'$ и $O'y'$ в системе Oxy ;
- ii. Координаты точки O и единичных векторов \vec{e}_1, \vec{e}_2 осей Ox и Oy в системе $O'x'y'$;
- iii. Координаты единичной точки E' в системе Oxy ;
- iv. Координаты единичной точки E в системе $O'x'y'$;
- v. Уравнения координатных осей $O'x'$ и $O'y'$ в системе Oxy ;
- vi. Уравнения координатных осей Ox и Oy в системе $O'x'y'$;

vii. Уравнения прямой ℓ_1 в системе $O'x'y'$;

viii. Уравнения прямой ℓ_2 в системе Oxy .

1. $x = 5x' + 4y' + 5, y = 2x' - 5y' - 2;$
 $\ell_1: x - 2y + 2 = 0; \quad \ell_2: 5x' - 2y' - 3 = 0.$

Ответ: 1), $O'(5, -2), \vec{i}' = (5, 2), \vec{j}' = (4, -5), 2) , O(-\frac{17}{33}, -\frac{20}{33}), \vec{i} = (\frac{5}{33}, \frac{2}{33}), \vec{j} = (\frac{4}{33}, -\frac{5}{33}), 3) , E'(14, -5), 4) , E(-\frac{8}{33}, -\frac{23}{33}), 5) , Ox' 2x - 5y - 20 = 0, Oy' 5x + 4y - 17 = 0, 6) , Ox 2x' - 5y' - 2 = 0, Oy 5x' + 4y' + 5 = 0, \ell'_1 x' + 14y' + 11 = 0, \ell'_2 \frac{10}{11}y - \frac{48}{11} + \frac{7}{11}x = 0.$

2. $x = 5x' - 2y' + 2, y = 4x' + 4y' - 1;$
 $\ell_1: x - 2y + 1 = 0; \quad \ell_2: 5x' - 5y' - 1 = 0.$

Ответ: 1), $O'(2, -1), \vec{i}' = (5, 4), \vec{j}' = (-2, 4), 2) , O(-3/14, \frac{13}{28}), \vec{i} = (1/7, -1/7), \vec{j} = (1/14, \frac{5}{28}), 3) , E'(5, 7), 4) , E = (0, 1/2), 5) , Ox' 4x - 5y - 13 = 0, Oy' - y + 3 - 2x = 0, 6) , Ox 4x' + 4y' - 1 = 0, Oy 5x' - 2y' + 2 = 0, \ell'_1 - 3x' - 10y' + 5 = 0, \ell'_2 - \frac{15}{28}y - \frac{123}{28} + \frac{10}{7}x = 0.$

3. $x = 4x' - 3y' - 3, y = 4x' - 2y' - 3;$
 $\ell_1: 2x + y + 1 = 0; \quad \ell_2: 5x' - 4y' - 3 = 0.$

Ответ: 1), $O'(-3, -3), \vec{i}' = (4, 4), \vec{j}' = (-3, -2), 2) , O(3/4, 1), \vec{i} = (-1/2, -1), \vec{j} = (3/4, 1), 3) , E'(-2, -1), 4) , E(1, 0), 5) , Ox' - y + x = 0, Oy' 2x - 3 - 3y = 0, 6) , Ox 4x' - 2y' - 3 = 0, Oy 4x' - 3y' - 3 = 0, \ell'_1 24x' - 16y' - 16 = 0, \ell'_2 3/2x + 3/4 - 1/4y = 0.$

4. $x = 4x' - 4y' + 1, y = 3x' - 2y' + 1;$
 $\ell_1: x + y + 1 = 0; \quad \ell_2: 5x' - 3y' + 3 = 0.$

Ответ: 1), $O'(1, 1), \vec{i}' = (4, 3), \vec{j}' = (-4, -2), 2) , O(-1/2, -1/4), \vec{i} = (-1/2, -3/4), \vec{j} = (1, 1), 3) , E'(1, 2), 4) , E(0, 0), 5) , Ox' 3x - 4y + 1 = 0, Oy' x - 2y + 1 = 0, 6) , Ox 3x' - 2y' + 1 = 0, Oy 4x' - 4y' + 1 = 0, \ell'_1 7x' - 6y' + 3 = 0, \ell'_2 - 1/4x + 2y + 5/4 = 0.$

5. $x = 5x' + 5y' + 2, y = 3x' - 2y' - 3;$
 $\ell_1: 4x - 5y + 4 = 0; \quad \ell_2: 5x' - 5y' - 3 = 0.$

Ответ: 1), $O'(2, -3), \vec{i}' = (5, 3), \vec{j}' = (5, -2), 2) , O(\frac{11}{25}, -\frac{21}{25}), \vec{i} = (\frac{2}{25}, \frac{3}{25}), \vec{j} = (1/5, -1/5), 3) , E'(12, -2), 4) , E(\frac{18}{25}, -\frac{23}{25}), 5) , Ox' 3x -$

$$-5y - 21 = 0, Oy' 2x + 11 + 5y = 0, 6), Ox 3x' - 2y' - 3 = 0, Oy 5x' + 5y' + 2 = 0, \ell'_1 5x' + 30y' + 27 = 0, \ell'_2 -1/5x + \frac{17}{5} + 2y = 0.$$

6. $x = 3x' + 4y' + 4, y = 5x' - 3y' - 4;$
 $\ell_1: 4x + 4y + 3 = 0; \ell_2: x' + 5y' - 1 = 0.$

Ответ: 1), $O'(4, -4), \vec{i}' = (3, 5), \vec{j}' = (4, -3), 2), O(\frac{4}{29}, -\frac{32}{29}), \vec{i} = (\frac{3}{29}, \frac{5}{29}), \vec{j} = (\frac{4}{29}, -\frac{3}{29}), 3), E'(11, -2), 4), E(\frac{11}{29}, -\frac{30}{29}), 5), Ox' - 3y' - 32 + 5x = 0, Oy' 3x + 4y + 4 = 0, 6), Ox 5x' - 3y' - 4 = 0, Oy 3x' + 4y' + 4 = 0, \ell'_1 32x' + 4y' + 3 = 0, \ell'_2 \frac{28}{29}x - \frac{11}{29}y - \frac{185}{29} = 0.$

7. $x = 3x' + 2y' + 5, y = 2x' + 4y' - 2;$
 $\ell_1: x + y + 2 = 0; \ell_2: 2x' + 3y' - 2 = 0.$

Ответ: 1), $O'(5, -2), \vec{i}' = (3, 2), \vec{j}' = (2, 4), 2), O(-3, 2), \vec{i} = (1/2, -1/4), \vec{j} = (-1/4, 3/8), 3), E'(10, 4), 4), E(-11/4, \frac{17}{8}), 5), Ox' 2x - 3y - 16 = 0, Oy' - y - 12 + 2x = 0, 6), Ox - 1 + x' + 2y' = 0, Oy 3x' + 2y' + 5 = 0, \ell'_1 10x' + 12y' + 10 = 0, \ell'_2 5/8y - 2 + 1/4x = 0.$

8. $x = 2x' - 4y' - 4, y = 5x' - 4y' - 4;$
 $\ell_1: 3x + 4y - 1 = 0; \ell_2: 5x' + 2y' + 3 = 0.$

Ответ: 1), $O'(-4, -4), \vec{i}' = (2, 5), \vec{j}' = (-4, -4), 2), O(1/3, -1), \vec{i} = (-1/3, -\frac{5}{12}), \vec{j} = (1/3, 1/6), 3), E'(-6, -3), 4), E(0, -5/4), 5), Ox' - 2y' + 12 + 5x = 0, Oy' - y + x = 0, 6), Ox 5x' - 4y' - 4 = 0, Oy x' - 2y' - 2 = 0, \ell'_1 26x' - 28y' - 29 = 0, \ell'_2 -5/2x + 2y + 1 = 0.$

9. $x = 3x' + 3y' + 3, y = 2x' - 5y' + 3;$
 $\ell_1: x + 5y - 3 = 0; \ell_2: 2x' + 5y' - 4 = 0.$

Ответ: 1), $O'(3, 3), \vec{i}' = (3, 2), \vec{j}' = (3, -5), 2), O(-\frac{8}{7}, 1/7), \vec{i} = (\frac{5}{21}, 2/21), \vec{j} = (1/7, -1/7), 3), E'(9, 0), 4), E(-\frac{16}{21}, 2/21), 5), Ox' - 3y' + 3 + 2x = 0, Oy' 3y - 24 + 5x = 0, 6), Ox 2x' - 5y' + 3 = 0, Oy x' + y' + 1 = 0, \ell'_1 13x' - 22y' + 15 = 0, \ell'_2 \frac{20}{21}x - 3/7y - \frac{39}{7} = 0.$

10. $x = 3x' - 3y' - 2, y = 3x' + 3y' + 5;$
 $\ell_1: 4x + 3y + 2 = 0; \ell_2: 3x' - 4y' - 5 = 0.$

Ответ: 1), $O'(-2, 5)$, $\vec{i}' = (3, 3)$, $\vec{j}' = (-3, 3)$, 2) , $O(-1/2, -7/6)$, $\vec{i}' = (1/6, -1/6)$, $\vec{j}' = (1/6, 1/6)$, 3) , $E'(-2, 11)$, 4) , $E(-1/6, -7/6)$, 5) , $Ox' - y + 7 + x = 0$, $Oy' - y + 3 - x = 0$, 6) , $Ox' 3x' + 3y' + 5 = 0$, $Oy' 3x' - 3y' - 2 = 0$, $\ell'_1 21x' - 3y' + 9 = 0$, $\ell'_2 7/6x' - 1/6y' - \frac{11}{6} = 0$.

11. $x = x' - 2y' + 4$, $y = 3x' + 4y' - 5$;
 $\ell_1: x - 2y - 1 = 0$; $\ell_2: 5x' - 4y' + 2 = 0$.

Ответ: 1), $O'(4, -5)$, $\vec{i}' = (1, 3)$, $\vec{j}' = (-2, 4)$, 2) , $O(-3/5, \frac{17}{10})$, $\vec{i}' = (2/5, -3/10)$, $\vec{j}' = (1/5, 1/10)$, 3) , $E'(3, 2)$, 4) , $E(0, 3/2)$, 5) , $Ox' - y + 3x - 17 = 0$, $Oy' - y + 3 - 2x = 0$, 6) , $Ox' 3x' + 4y' - 5 = 0$, $Oy' x' - 2y' + 4 = 0$, $\ell'_1 - 10x' - 20y' + 26 = 0$, $\ell'_2 \frac{16}{5}x' + 3/5y' - \frac{39}{5} = 0$.

12. $x = 2x' - 5y' + 4$, $y = 4x' - 3y' + 2$;
 $\ell_1: 5x + 5y - 1 = 0$; $\ell_2: 3x' - 2y' + 1 = 0$.

Ответ: 1), $O'(4, 2)$, $\vec{i}' = (2, 4)$, $\vec{j}' = (-5, -3)$, 2) , $O(1/7, 6/7)$, $\vec{i}' = (-3/14, -2/7)$, $\vec{j}' = (\frac{5}{14}, 1/7)$, 3) , $E'(1, 3)$, 4) , $E(2/7, 5/7)$, 5) , $Ox' - y - 6 + 2x = 0$, $Oy' 3x - 5y - 2 = 0$, 6) , $Ox' 4x' - 3y' + 2 = 0$, $Oy' 2x' - 5y' + 4 = 0$, $\ell'_1 30x' - 40y' + 29 = 0$, $\ell'_2 - 1/14x' + \frac{11}{14}y' - 2/7 = 0$.

13. $x = 3x' + y' - 2$, $y = x' - 2y' + 3$;
 $\ell_1: x + y + 2 = 0$; $\ell_2: 4x' - y' + 1 = 0$.

Ответ: 1), $O'(-2, 3)$, $\vec{i}' = (3, 1)$, $\vec{j}' = (1, -2)$, 2) , $O(1/7, \frac{11}{7})$, $\vec{i}' = (2/7, 1/7)$, $\vec{j}' = (1/7, -3/7)$, 3) , $E'(2, 2)$, 4) , $E(4/7, \frac{9}{7})$, 5) , $Ox' - 3y + x + 11 = 0$, $Oy' y + 2x + 1 = 0$, 6) , $Ox' x' - 2y' + 3 = 0$, $Oy' 3x' + y' - 2 = 0$, $\ell'_1 4x' - y' + 3 = 0$, $\ell'_2 y + x = 0$.

14. $x = 5x' + y' + 5$, $y = 5x' + 4y' - 2$;
 $\ell_1: 3x - 5y + 2 = 0$; $\ell_2: 5x' - y' - 4 = 0$.

Ответ: 1), $O'(5, -2)$, $\vec{i}' = (5, 5)$, $\vec{j}' = (1, 4)$, 2) , $O(-\frac{22}{15}, 7/3)$, $\vec{i}' = (\frac{4}{15}, -1/3)$, $\vec{j}' = (-1/15, 1/3)$, 3) , $E'(11, 7)$, 4) , $E(-\frac{19}{15}, 7/3)$, 5) , $Ox' - y + x - 7 = 0$, $Oy' 4x - y - 22 = 0$, 6) , $Ox' 5x' + 4y' - 2 = 0$, $Oy' 5x' + y' + 5 = 0$, $\ell'_1 - 10x' - 17y' + 27 = 0$, $\ell'_2 - 2/3y' + 5/3x' - \frac{41}{3} = 0$.

15. $x = 4x' - 4y' + 5, y = 2x' + y' + 5;$
 $\ell_1: 5x + 2y + 2 = 0; \ell_2: 4x' - 5y' + 1 = 0.$

Ответ: 1), $O'(5, 5), \vec{i}' = (4, 2), \vec{j}' = (-4, 1), 2), O(-\frac{25}{12}, -5/6), \vec{i} = (1/12, -1/6), \vec{j} = (1/3, 1/3), 3), E'(5, 8), 4), E(-5/3, -2/3), 5), Ox' - 2y' + 5 + x = 0, Oy' x + 4y - 25 = 0, 6), Ox 2x' + y' + 5 = 0, Oy 4x' - 4y' + 5 = 0, \ell'_1 24x' - 18y' + 37 = 0, \ell'_2 7/6x - 1/3y - \frac{19}{6} = 0.$

16. $x = 5x' - 4y' - 5, y = 3x' - y' - 4;$
 $\ell_1: 5x - y + 3 = 0; \ell_2: 2x' - 2y' - 1 = 0.$

Ответ: 1), $O'(-5, -4), \vec{i}' = (5, 3), \vec{j}' = (-4, -1), 2), O(\frac{11}{7}, 5/7), \vec{i} = (-1/7, -3/7), \vec{j} = (4/7, 5/7), 3), E'(-4, -2), 4), E(2, 1), 5), Ox' 3x - 5y - 5 = 0, Oy' x - 4y - 11 = 0, 6), Ox 3x' - y' - 4 = 0, Oy 5x' - 4y' - 5 = 0, \ell'_1 22x' - 19y' - 18 = 0, \ell'_2 4/7x - 2/7y + 5/7 = 0.$

17. $x = 3x' + 4y' - 5, y = x' - 4y' - 4;$
 $\ell_1: x - y + 3 = 0; \ell_2: 5x' - 4y' + 2 = 0.$

Ответ: 1), $O'(-5, -4), \vec{i}' = (3, 1), \vec{j}' = (4, -4), 2), O(9/4, -\frac{7}{16}), \vec{i} = (1/4, 1/16), \vec{j} = (1/4, -3/16), 3), E'(2, -7), 4), E(11/4, -\frac{9}{16}), 5), Ox' - 3y - 7 + x = 0, Oy' x + y + 9 = 0, 6), Ox x' - 4y' - 4 = 0, Oy 3x' + 4y' - 5 = 0, \ell'_1 2x' + 8y' + 2 = 0, \ell'_2 x + 2y + 15 = 0.$

18. $x = 5x' - 2y' + 4, y = x' + 4y' + 1;$
 $\ell_1: 4x + 5y - 3 = 0; \ell_2: 3x' + 3y' - 1 = 0.$

Ответ: 1), $O'(4, 1), \vec{i}' = (5, 1), \vec{j}' = (-2, 4), 2), O(-\frac{9}{11}, -1/22), \vec{i} = (2/11, -1/22), \vec{j} = (1/11, \frac{5}{22}), 3), E'(7, 6), 4), E = (-\frac{6}{11}, \frac{3}{22}), 5), Ox' x - 5y + 1 = 0, Oy' y - 9 + 2x = 0, 6), Ox x' + 4y' + 1 = 0, Oy 5x' - 2y' + 4 = 0, \ell'_1 25x' + 12y' + 18 = 0, \ell'_2 \frac{21}{22}y - \frac{79}{22} + \frac{9}{22}x = 0.$

19. $x = 3x' + 2y' + 3, y = 2x' - 2y' + 3;$
 $\ell_1: x - y + 4 = 0; \ell_2: 3x' - y' + 3 = 0.$

Ответ: 1), $O'(3, 3), \vec{i}' = (3, 2), \vec{j}' = (2, -2), 2), O(-6/5, 3/10), \vec{i} = (1/5, 1/5), \vec{j} = (1/5, -3/10), 3), E'(8, 3), 4), E(-4/5, 1/5), 5), Ox' - 3y + 2x + 3 = 0, Oy' x + y - 6 = 0, 6), Ox 2x' - 2y' + 3 = 0, Oy 3x' + 2y' + 3 = 0, \ell'_1 x' + 4y' + 4 = 0, \ell'_2 \frac{9}{10}y - \frac{9}{10} + 2/5x = 0.$

20. $x = 5x' + 2y' - 4$, $y = 5x' - 5y' + 3$;
 $\ell_1: 5x - 3y - 2 = 0$; $\ell_2: 5x' - y' - 5 = 0$.

Ответ: 1), $O'(-4, 3)$, $\vec{i}' = (5, 5)$, $\vec{j}' = (2, -5)$, 2), $O(2/5, 1)$, $\vec{i} = (1/7, 1/7)$, $\vec{j} = (\frac{2}{35}, -1/7)$, 3), $E'(3, 3)$, 4), $E(3/5, 1)$, 5), $Ox' - y + x + 7 = 0$, $Oy' 2y + 14 + 5x = 0$, 6), $Ox 5x' - 5y' + 3 = 0$, $Oy 5x' + 2y' - 4 = 0$, $\ell'_1 10x' + 25y' - 31 = 0$, $\ell'_2 3/7y - 4 + 4/7x = 0$.

21. $x = x' + y' + 2$, $y = 4x' + 3y' - 4$;
 $\ell_1: 4x + 5y - 5 = 0$; $\ell_2: 5x' + y' + 5 = 0$.

Ответ: 1), $O'(2, -4)$, $\vec{i}' = (1, 4)$, $\vec{j}' = (1, 3)$, 2), $O(10, -12)$, $\vec{i} = (-3, 4)$, $\vec{j} = (1, -1)$, 3), $E'(4, 3)$, 4), $E(8, -9)$, 5), $Ox' - y + 4x - 12 = 0$, $Oy' 3x - y - 10 = 0$, 6), $Ox 4x' + 3y' - 4 = 0$, $Oy x' + y' + 2 = 0$, $\ell'_1 24x' + 19y' - 17 = 0$, $\ell'_2 - 11x + 4y + 43 = 0$.

22. $x = 4x' + y' + 4$, $y = 2x' - 3y' - 4$;
 $\ell_1: 5x - 5y + 4 = 0$; $\ell_2: 2x' + y' + 4 = 0$.

Ответ: 1), $O'(4, -4)$, $\vec{i}' = (4, 2)$, $\vec{j}' = (1, -3)$, 2), $O(-4/7, -\frac{12}{7})$, $\vec{i} = (3/14, 1/7)$, $\vec{j} = (1/14, -2/7)$, 3), $E'(9, -5)$, 4), $E(-2/7, -\frac{13}{7})$, 5), $Ox' - 2y + x - 12 = 0$, $Oy' y + 3x - 8 = 0$, 6), $Ox 2x' - 3y' - 4 = 0$, $Oy 4x' + y' + 4 = 0$, $\ell'_1 10x' + 20y' + 44 = 0$, $\ell'_2 - 1/7y + 4/7x + \frac{8}{7} = 0$.

23. $x = 2x' - 5y' + 5$, $y = 5x' - 4y' - 2$;
 $\ell_1: 4x - 5y + 4 = 0$; $\ell_2: 3x' - 5y' + 3 = 0$.

Ответ: 1), $O'(5, -2)$, $\vec{i}' = (2, 5)$, $\vec{j}' = (-5, -4)$, 2), $O(\frac{30}{17}, \frac{29}{17})$, $\vec{i} = (-\frac{4}{17}, -\frac{5}{17})$, $\vec{j} = (\frac{5}{17}, 2/17)$, 3), $E'(2, -1)$, 4), $E(\frac{31}{17}, \frac{26}{17})$, 5), $Ox' - 2y + 5x - 29 = 0$, $Oy' 4x - 30 - 5y = 0$, 6), $Ox 5x' - 4y' - 2 = 0$, $Oy 2x' - 5y' + 5 = 0$, $\ell'_1 - 17x' + 34 = 0$, $\ell'_2 \frac{13}{17}x - \frac{4}{17} + \frac{5}{17}y = 0$.

24. $x = x' + 3y' + 3$, $y = 4x' - 5y' - 1$;
 $\ell_1: 3x + y - 5 = 0$; $\ell_2: 2x' + y' - 2 = 0$.

Ответ: 1), $O'(3, -1)$, $\vec{i}' = (1, 4)$, $\vec{j}' = (3, -5)$, 2), $O(-\frac{12}{17}, -\frac{13}{17})$, $\vec{i} = (\frac{5}{17}, \frac{4}{17})$, $\vec{j} = (\frac{3}{17}, -1/17)$, 3), $E'(7, -2)$, 4), $E(-\frac{4}{17}, -\frac{10}{17})$, 5), $Ox' -$

$$-y + 4x - 13 = 0, Oy' \ 3y + 5x - 12 = 0, 6), Ox \ 4x' - 5y' - 1 = 0, Oy \ x' + 3y' + 3 = 0, \ell'_1 \ 7x' + 4y' + 3 = 0, \ell'_2 \ \frac{5}{17}y + \frac{14}{17}x - \frac{71}{17} = 0.$$

25. $x = x' - 4y' + 2, y = 4x' - 4y' + 4;$
 $\ell_1: 3x - 2y - 2 = 0; \quad \ell_2: 3x' - 3y' - 3 = 0.$

Ответ: 1), $O'(2, 4), \vec{i}' = (1, 4), \vec{j}' = (-4, -4), 2), O(-2/3, 1/3), \vec{i}' = (-1/3, -1/3), \vec{j}' = (1/3, 1/12), 3), E'(-1, 4), 4), E(-2/3, 1/12), 5), Ox' - y + 4x - 4 = 0, Oy' \ x + 2 - y = 0, 6), Ox \ 1 + x' - y' = 0, Oy \ x' - 4y' + 2 = 0, \ell'_1 \ -5x' - 4y' - 4 = 0, \ell'_2 \ 3/4y - 6 = 0.$

26. $x = 3x' - 4y' - 5, y = 2x' + 5y' - 4;$
 $\ell_1: 5x + 3y - 5 = 0; \quad \ell_2: 4x' - y' + 3 = 0.$

Ответ: 1), $O'(-5, -4), \vec{i}' = (3, 2), \vec{j}' = (-4, 5), 2), O(\frac{41}{23}, 2/23), \vec{i}' = (\frac{5}{23}, -2/23), \vec{j}' = (\frac{4}{23}, \frac{3}{23}), 3), E'(-6, 3), 4), E(\frac{50}{23}, \frac{3}{23}), 5), Ox' - 3y - 2 + 2x = 0, Oy' \ 5x + 4y + 41 = 0, 6), Ox \ 2x' + 5y' - 4 = 0, Oy \ 3x' - 4y' - 5 = 0, \ell'_1 \ 21x' - 5y' - 42 = 0, \ell'_2 \ \frac{22}{23}x + \frac{13}{23}y + \frac{231}{23} = 0.$

27. $x = 2x' + 3y' - 4, y = 5x' + y' - 4;$
 $\ell_1: 5x - 3y + 3 = 0; \quad \ell_2: 4x' - 2y' - 1 = 0.$

Ответ: 1), $O'(-4, -4), \vec{i}' = (2, 5), \vec{j}' = (3, 1), 2), O(\frac{8}{13}, \frac{12}{13}), \vec{i}' = (-1/13, \frac{5}{13}), \vec{j}' = (3/13, -2/13), 3), E'(1, 2), 4), E(\frac{10}{13}, \frac{15}{13}), 5), Ox' - 2y + 12 + 5x = 0, Oy' \ x - 3y - 8 = 0, 6), Ox \ 5x' + y' - 4 = 0, Oy \ 2x' + 3y' - 4 = 0, \ell'_1 \ -5x' + 12y' - 5 = 0, \ell'_2 \ -\frac{14}{13}x + \frac{16}{13}y - \frac{5}{13} = 0.$

28. $x = 3x' + 2y' - 3, y = 3x' + 5y' + 3;$
 $\ell_1: 2x + y - 4 = 0; \quad \ell_2: 3x' + 3y' + 5 = 0.$

Ответ: 1), $O'(-3, 3), \vec{i}' = (3, 3), \vec{j}' = (2, 5), 2), O(7/3, -2), \vec{i}' = (5/9, -1/3), \vec{j}' = (-2/9, 1/3), 3), E'(2, 11), 4), E(8/3, -2), 5), Ox' - y + x + 6 = 0, Oy' \ -2y + 5x + 21 = 0, 6), Ox \ 3x' + 5y' + 3 = 0, Oy \ 3x' + 2y' - 3 = 0, \ell'_1 \ 9x' + 9y' - 7 = 0, \ell'_2 \ 1/3y + 2/3x + 6 = 0.$

29. $x = 2x' + 5y' + 3, y = 2x' - 5y' + 1;$
 $\ell_1: 3x - y - 5 = 0; \quad \ell_2: 2x' - 3y' + 1 = 0.$

Ответ: 1), $O'(3, 1), \vec{i}' = (2, 2), \vec{j}' = (5, -5), 2), O(-1, -1/5), \vec{i}' = (1/4, 1/10), \vec{j}' = (1/4, -1/10), 3), E'(10, -2), 4), E(-1/2, -1/5), 5), Ox'$

$$-y - 2 + x = 0, Oy' x + y - 4 = 0, 6), Ox 2x' - 5y' + 1 = 0, Oy 2x' + 5y' + 3 = 0, \ell'_1 4x' + 20y' + 3 = 0, \ell'_2 1/5x + 4/5y - 2/5 = 0.$$

30. $x = 2x' - 2y' - 5, y = 4x' + 5y' - 5;$
 $\ell_1: 2x + 3y + 3 = 0; \ell_2: 4x' - y' + 4 = 0.$

Ответ: 1), $O'(-5, -5), \vec{i}' = (2, 4), \vec{j}' = (-2, 5), 2), O(\frac{35}{18}, -5/9), \vec{i}' = (\frac{5}{18}, -2/9), \vec{j}' = (1/9, 1/9), 3), E'(-5, 4), 4), E(7/3, -2/3), 5), Ox' - y + 2x + 5 = 0, Oy' 5x + 35 + 2y = 0, 6), Ox 4x' + 5y' - 5 = 0, Oy 2x' - 2y' - 5 = 0, \ell'_1 16x' + 11y' - 22 = 0, \ell'_2 1/3y + 4/3x + \frac{37}{3} = 0.$

Задача 1.10. Написать формулы преобразования координат на плоскости, принимая за новые координатные оси Ox' и Oy' соответственно прямые ℓ_1 и ℓ_2 , а за за единичную точку новой системы координат — точку E' .

1. $\ell_1: x - 4y - 17 = 0, \ell_2: 3x + 2y - 9 = 0; E'(-4, 5).$

Ответ: $(x' = -3/11x - 2/11y + \frac{9}{11}, y' = -1/41x + \frac{4}{41}y + \frac{17}{41}), (x = -\frac{22}{7}x' - \frac{41}{7}y' + 5, y = -\frac{11}{14}x' + \frac{123}{14}y' - 3).$

2. $\ell_1: 2x + 3y + 16 = 0, \ell_2: 3x - 5y - 14 = 0; E'(-3, 1).$

Ответ: $(x' = -\frac{3}{28}x + \frac{5}{28}y + 1/2, y' = 2/13x + 3/13y + \frac{16}{13}), (x = -\frac{84}{19}x' + \frac{65}{19}y' - 2, y = \frac{56}{19}x' + \frac{39}{19}y' - 4).$

3. $\ell_1: x - y + 5 = 0, \ell_2: 2x - 5y + 16 = 0; E'(1, 1).$

Ответ: $(x' = 2/13x - \frac{5}{13}y + \frac{16}{13}, y' = 1/5x - 1/5y + 1), (x = -13/3x' + \frac{25}{3}y' - 3, y = -13/3x' + 10/3y' + 2).$

4. $\ell_1: x - 4y + 3 = 0, \ell_2: 4x + y - 5 = 0; E'(4, 5).$

Ответ: $(x' = 1/4x + 1/16y - \frac{5}{16}, y' = -1/13x + \frac{4}{13}y - 3/13), (x = \frac{64}{17}x' - \frac{13}{17}y' + 1, y = \frac{16}{17}x' + \frac{52}{17}y' + 1).$

5. $\ell_1: x - 2y + 7 = 0, \ell_2: 2x - y + 5 = 0; E'(3, -4).$

Ответ: $(x' = 2/15x - 1/15y + 1/3, y' = 1/18x - 1/9y + \frac{7}{18}), (x = 10x' - 6y' - 1, y = 5x' - 12y' + 3).$

6. $\ell_1: 4x - 3y - 26 = 0, \ell_2: x + 3y + 1 = 0; E'(4, -5).$

Ответ: $(x' = -1/10 x - 3/10 y - 1/10, y' = 4/5 x - 3/5 y - \frac{26}{5}), (x = -2x' + y' + 5, y = -8/3 x' - 1/3 y' - 2).$

7. $\ell_1: 4x - y - 17 = 0, \ell_2: 5x - 2y - 19 = 0; E'(4, 5).$

Ответ: $(x' = -5/9 x + 2/9 y + \frac{19}{9}, y' = -2/3 x + 1/6 y + \frac{17}{6}), (x = 3x' - 4y' + 5, y = 12x' - 10y' + 3).$

8. $\ell_1: x - y = 0, \ell_2: 5x + y + 18 = 0; E'(-5, -2).$

Ответ: $(x' = -5/9 x - 1/9 y - 2, y' = -1/3 x + 1/3 y), (x = -3/2 x' - 1/2 y' - 3, y = -3/2 x' + 5/2 y' - 3).$

9. $\ell_1: x + 3y - 2 = 0, \ell_2: 5x - 2y + 24 = 0; E'(2, -4).$

Ответ: $(x' = \frac{5}{42} x - 1/21 y + 4/7, y' = -1/12 x - 1/4 y + 1/6), (x = \frac{126}{17} x' - \frac{24}{17} y' - 4, y = -\frac{42}{17} x' - \frac{60}{17} y' + 2).$

10. $\ell_1: x + y + 9 = 0, \ell_2: 4x - 3y + 1 = 0; E'(2, -1).$

Ответ: $(x' = 1/3 x - 1/4 y + 1/12, y' = 1/10 x + 1/10 y + \frac{9}{10}), (x = \frac{12}{7} x' + \frac{30}{7} y' - 4, y = -\frac{12}{7} x' + \frac{40}{7} y' - 5).$

11. $\ell_1: 2x - y + 11 = 0, \ell_2: x - 3y + 13 = 0; E'(2, 4).$

Ответ: $(x' = 1/3 x - y + 13/3, y' = 2/11 x - 1/11 y + 1), (x = -3/5 x' + \frac{33}{5} y' - 4, y = -6/5 x' + \frac{11}{5} y' + 3).$

12. $\ell_1: 2x + y + 8 = 0, \ell_2: 4x - y + 22 = 0; E'(-2, -5).$

Ответ: $(x' = \frac{4}{19} x - 1/19 y + \frac{22}{19}, y' = -2x - y - 8), (x = \frac{19}{6} x' - 1/6 y' - 5, y = -\frac{19}{3} x' - 2/3 y' + 2).$

13. $\ell_1: 5x - 2y + 10 = 0, \ell_2: 2x - 3y - 7 = 0; E'(-4, -1).$

Ответ: $(x' = -1/6 x + 1/4 y + \frac{7}{12}, y' = -5/8 x + 1/4 y - 5/4), (x = \frac{24}{11} x' - \frac{24}{11} y' - 4, y = \frac{60}{11} x' - \frac{16}{11} y' - 5).$

14. $\ell_1: 2x - 5y + 14 = 0, \ell_2: 2x - y - 2 = 0; E'(-4, 5).$

Ответ: $(x' = -2/15 x + 1/15 y + 2/15, y' = -2/19 x + \frac{5}{19} y - \frac{14}{19}), (x = -\frac{75}{8} x' + \frac{19}{8} y' + 3, y = -\frac{15}{4} x' + \frac{19}{4} y' + 4).$

15. $\ell_1: x + y - 3 = 0, \ell_2: 3x + 5y - 17 = 0; E'(5, 1).$

Ответ: $(x' = x + 5/3 y - \frac{17}{3}, y' = 1/3 x + 1/3 y - 1), (x = -3/2 x' + 15/2 y' - 1, y = 3/2 x' - 9/2 y' + 4).$

16. $l_1: 2x - 5y + 31 = 0, l_2: 4x + 5y - 13 = 0; E'(5, 4).$

ОТВЕТ: $(x' = \frac{4}{27}x + \frac{5}{27}y - \frac{13}{27}, y' = 2/21x - \frac{5}{21}y + \frac{31}{21}), (x = 9/2x' + 7/2y' - 3, y = 9/5x' - \frac{14}{5}y' + 5).$

17. $l_1: x + 3y + 9 = 0, l_2: x + 5y + 17 = 0; E'(-1, 3).$

ОТВЕТ: $(x' = 1/31x + \frac{5}{31}y + \frac{17}{31}, y' = 1/17x + \frac{3}{17}y + \frac{9}{17}), (x = -\frac{93}{2}x' + \frac{85}{2}y' + 3, y = \frac{31}{2}x' - 17/2y' - 4).$

18. $l_1: 2x + 5y - 15 = 0, l_2: x + 3y - 10 = 0; E'(-2, 2).$

ОТВЕТ: $(x' = -1/6x - 1/2y + 5/3, y' = -2/9x - 5/9y + 5/3), (x = 30x' - 27y' - 5, y = -12x' + 9y' + 5).$

19. $l_1: x - y + 1 = 0, l_2: x + 2y - 14 = 0; E'(3, -2).$

ОТВЕТ: $(x' = -1/15x - 2/15y + \frac{14}{15}, y' = 1/6x - 1/6y + 1/6), (x = -5x' + 4y' + 4, y = -5x' - 2y' + 5).$

20. $l_1: 3x + 4y - 15 = 0, l_2: 3x + 2y - 9 = 0; E'(-1, -4).$

ОТВЕТ: $(x' = -\frac{3}{20}x - 1/10y + \frac{9}{20}, y' = -\frac{3}{34}x - 2/17y + \frac{15}{34}), (x = -\frac{40}{3}x' + \frac{34}{3}y' + 1, y = 10x' - 17y' + 3).$

21. $l_1: 5x + y + 8 = 0, l_2: x - y - 2 = 0; E'(3, -5).$

ОТВЕТ: $(x' = 1/6x - 1/6y - 1/3, y' = \frac{5}{18}x + 1/18y + 4/9), (x = x' + 3y' - 1, y = -5x' + 3y' - 3).$

22. $l_1: 5x - 4y + 32 = 0, l_2: 3x + y + 9 = 0; E'(-2, -4).$

ОТВЕТ: $(x' = -3x - y - 9, y' = \frac{5}{38}x - 2/19y + \frac{16}{19}), (x = -\frac{4}{17}x' + \frac{38}{17}y' - 4, y = -\frac{5}{17}x' - \frac{114}{17}y' + 3).$

23. $l_1: x + 2y - 7 = 0, l_2: x - y - 1 = 0; E'(-1, 5).$

ОТВЕТ: $(x' = -1/7x + 1/7y + 1/7, y' = 1/2x + y - 7/2), (x = -14/3x' + 2/3y' + 3, y = 7/3x' + 2/3y' + 2).$

24. $l_1: 2x + 3y - 11 = 0, l_2: 2x + 5y - 17 = 0; E'(2, -5).$

ОТВЕТ: $(x' = -1/19x - \frac{5}{38}y + \frac{17}{38}, y' = -1/11x - \frac{3}{22}y + 1/2), (x = \frac{57}{2}x' - \frac{55}{2}y' + 1, y = -19x' + 11y' + 3).$

25. $l_1: 3x - 5y - 8 = 0, l_2: x + 4y + 3 = 0; E'(-1, -2).$

ОТВЕТ: $(x' = -1/6x - 2/3y - 1/2, y' = -3x + 5y + 8), (x = -\frac{30}{17}x' - \frac{4}{17}y' + 1, y = -\frac{18}{17}x' + 1/17y' - 1).$

26. $\ell_1: x - y + 3 = 0$, $\ell_2: 3x + y - 11 = 0$; $E'(-4, -4)$.

Ответ: $(x' = -1/9x - 1/27y + \frac{11}{27}, y' = 1/3x - 1/3y + 1)$, $(x = -\frac{27}{4}x' + 3/4y' + 2, y = -\frac{27}{4}x' - 9/4y' + 5)$.

27. $\ell_1: 2x + 3y + 14 = 0$, $\ell_2: 3x - 4y - 13 = 0$; $E'(-4, -3)$.

Ответ: $(x' = -3/13x + \frac{4}{13}y + 1, y' = -2/3x - y - 14/3)$, $(x = -\frac{39}{17}x' - \frac{12}{17}y' - 1, y = \frac{26}{17}x' - \frac{9}{17}y' - 4)$.

28. $\ell_1: x + 3y - 17 = 0$, $\ell_2: x + 2y - 12 = 0$; $E'(-1, 5)$.

Ответ: $(x' = -1/3x - 2/3y + 4, y' = -1/3x - y + \frac{17}{3})$, $(x = -9x' + 6y' + 2, y = 3x' - 3y' + 5)$.

29. $\ell_1: 3x - 5y - 11 = 0$, $\ell_2: x - 4y - 13 = 0$; $E'(5, -4)$.

Ответ: $(x' = 1/8x - 1/2y - \frac{13}{8}, y' = 1/8x - \frac{5}{24}y - \frac{11}{24})$, $(x = -\frac{40}{7}x' + \frac{96}{7}y' - 3, y = -\frac{24}{7}x' + \frac{24}{7}y' - 4)$.

30. $\ell_1: x - y - 3 = 0$, $\ell_2: 3x - y + 1 = 0$; $E'(-3, -5)$.

Ответ: $(x' = -x + 1/3y - 1/3, y' = -x + y + 3)$, $(x = -3/2x' + 1/2y' - 2, y = -3/2x' + 3/2y' - 5)$.

Задача 1.11. В пространстве заданы две системы координат $Oxyz$ и $O'x'y'z'$, связанные заданными соотношениями. Найти:

- i. Координаты начала системы $O'x'y'z'$ в системе $Oxyz$;
- ii. Канонические уравнения осей координат системы $O'x'y'z'$ в системе $Oxyz$;
- iii. Канонические уравнения осей координат системы $Oxyz$ в системе $O'x'y'z'$;
- iv. Уравнения координатных плоскостей системы $O'x'y'z'$ в системе $Oxyz$;
- v. Уравнение плоскости \mathcal{P} в системе $O'x'y'z'$;
- vi. Канонические уравнения прямой ℓ в системе $O'x'y'z'$.

$$1. \begin{cases} x = 5x' - y' - 3z' + 1, \\ y = x' - y' - 2z' + 1, \\ z = 3x' - 2y' - 4z' - 3, \\ \mathcal{P}: 2x + y - 2z + 1 = 0, \end{cases}$$

$$\ell: \frac{x+10}{4} = \frac{y+10}{-10} = \frac{-6+z}{4}.$$

Ответ: 1) $O'[x = 1, y = 1, z = -3]$,

$$2) \quad O'x', \frac{x-1}{5} = \frac{y-1}{1} = \frac{z+3}{3},$$

$$O'y', \frac{x-1}{-1} = \frac{y-1}{-1} = \frac{z+3}{-2},$$

$$O'z', \frac{x-1}{-3} = \frac{y-1}{-2} = \frac{z+3}{-4},$$

$$3) \quad Ox : \frac{x'-5}{0} = \frac{y'+34}{2} = \frac{z'-20}{-1},$$

$$Oy : \frac{x'-5}{-2} = \frac{y'+34}{11} = \frac{z'-20}{-7},$$

$$Oz : \frac{x'-5}{1} = \frac{y'+34}{-7} = \frac{z'-20}{4},$$

$$4) \quad O'x'y', -x - 7y + 4z + 20 = 0,$$

$$O'x'z' : 2x + 11y - 7z - 34 = 0,$$

$$O'y'z' : -2y + z + 5 = 0,$$

$$5) \quad \mathcal{P}' : 5x' + y' + 10 = 0,$$

$$6) \quad \ell' : \frac{x'-31}{-24} = \frac{y'+206}{130} = \frac{z'-124}{-82}.$$

$$2. \quad \begin{cases} x = -2x' - 6y' - 9z' - 3, \\ y = 2x' + 5y' + 8z' + 2, \\ z = -3x' - 9y' - 14z', \end{cases}$$

$$\mathcal{P} : x - 2y - z - 1 = 0,$$

$$\ell : \frac{x+9}{0} = \frac{y-5}{-8} = \frac{z-10}{2}.$$

$$\text{Ответ: } 1) \quad O'[x = -3, y = 2, z = 0],$$

$$2) \quad O'x', \frac{x+3}{-2} = \frac{y-2}{2} = \frac{z}{-3},$$

$$O'y', \frac{x+3}{-6} = \frac{y-2}{5} = \frac{z}{-9},$$

$$O'z', \frac{x+3}{-9} = \frac{y-2}{8} = \frac{z}{-14},$$

$$3) \quad Ox : \frac{x'+12}{-2} = \frac{y'+10}{-4} = \frac{z'-9}{3},$$

$$Oy : \frac{x'+12}{3} = \frac{y'+10}{-1} = \frac{z'-9}{0},$$

$$Oz : \frac{x'+12}{3} = \frac{y'+10}{2} = \frac{z'-9}{-2},$$

$$4) \quad O'x'y', 3x - 2z + 9 = 0,$$

$$O'x'z' : -4x - y + 2z - 10 = 0,$$

$$O'y'z' : -2x + 3y + 3z - 12 = 0,$$

$$5) \quad \mathcal{P}' : -3x' - 7y' - 11z' - 8 = 0,$$

$$6) \quad \ell' : \frac{x'-51}{18} = \frac{y'-41}{-12} = \frac{z'+38}{4}.$$

$$3. \quad \begin{cases} x = -x' - y' - 2z' - 2, \\ y = -x' - 2y' - 2z', \\ z = x' + y' + 3z' + 3, \end{cases}$$

$$\mathcal{P} : x - 4y - z + 1 = 0,$$

$$\ell : \frac{x-2}{-9} = \frac{y-3}{7} = \frac{z-8}{6}.$$

$$\text{Ответ: } 1) \quad O'[x = -2, y = 0, z = 3],$$

$$2) \quad O'x', \frac{x+2}{-1} = \frac{y}{-1} = \frac{z-3}{1},$$

$$O'y', \frac{x+2}{-1} = \frac{y}{-2} = \frac{z-3}{1},$$

$$O'z', \frac{x+2}{-2} = \frac{y}{-2} = \frac{z-3}{3},$$

$$3) \quad Ox : \frac{x'+2}{-4} = \frac{y'-2}{1} = \frac{z'+1}{1},$$

$$Oy : \frac{x'+2}{1} = \frac{y'-2}{-1} = \frac{z'+1}{0},$$

$$Oz : , \frac{x' + 2}{-2} = \frac{y' - 2}{0} = \frac{z' + 1}{1},$$

$$4) O'x'y', x + z - 1 = 0,$$

$$O'x'z' : , x - y + 2 = 0,$$

$$O'y'z' : -4x + y - 2z - 2 = 0,$$

$$5) \mathcal{P}' : 2x' + 6y' + 3z' - 4 = 0,$$

$$6) \ell' : \frac{x' + 23}{-31} = \frac{y' - 1}{16} = \frac{z' - 9}{3}.$$

$$4. \begin{cases} x = 2x' - 3y' + z' + 2, \\ y = -5x' + 8y' - 2z' - 1, \\ z = x' - 2y' + z', \end{cases}$$

$$\mathcal{P} : 3x - 3y - 2z - 4 = 0,$$

$$\ell : \frac{x + 10}{-8} = \frac{y + 10}{9} = \frac{z + 3}{-6}.$$

$$\text{Ответ: 1) } O'[x = 2, y = -1, z = 0],$$

$$2) , O'x', \frac{x - 2}{2} = \frac{y + 1}{-5} = \frac{z}{1},$$

$$O'y', \frac{x - 2}{-3} = \frac{y + 1}{8} = \frac{z}{-2},$$

$$O'z', \frac{x - 2}{1} = \frac{y + 1}{-2} = \frac{z}{1},$$

$$3) Ox : \frac{x' + 7}{4} = \frac{y' + 5}{3} = \frac{z' + 3}{2},$$

$$Oy : , \frac{x' + 7}{1} = \frac{y' + 5}{1} = \frac{z' + 3}{1},$$

$$Oz : , \frac{x' + 7}{-2} = \frac{y' + 5}{-1} = \frac{z' + 3}{1},$$

$$4) O'x'y', 2x + y + z - 3 = 0,$$

$$O'x'z' : , 3x + y - z - 5 = 0,$$

$$O'y'z' : 4x + y - 2z - 7 = 0,$$

$$5) \mathcal{P}' : 19x' - 29y' + 7z' + 5 = 0,$$

$$6) \ell' : \frac{x' + 51}{11} = \frac{y' + 42}{9} = \frac{z' + 36}{13}.$$

$$5. \begin{cases} x = -4x' + 2y' - z', \\ y = 5x' - 5y' + 6z', \\ z = -x' + 2y' - 3z' + 3, \end{cases}$$

$$\mathcal{P} : 4x - 2y - 4z + 5 = 0,$$

$$\ell : \frac{x - 9}{5} = \frac{y + 7}{8} = \frac{z - 10}{6}.$$

$$\text{Ответ: 1) } O'[x = 0, y = 0, z = 3],$$

$$2) , O'x', \frac{x}{-4} = \frac{y}{5} = \frac{z - 3}{-1},$$

$$O'y', \frac{x}{2} = \frac{y}{-5} = \frac{z - 3}{2},$$

$$O'z', \frac{x}{-1} = \frac{y}{6} = \frac{z - 3}{-3},$$

$$3) Ox : \frac{x' + 21}{3} = \frac{y' + 57}{9} = \frac{z' + 30}{5},$$

$$Oy : , \frac{x' + 21}{4} = \frac{y' + 57}{11} = \frac{z' + 30}{6},$$

$$Oz : , \frac{x' + 21}{7} = \frac{y' + 57}{19} = \frac{z' + 30}{10},$$

$$4) O'x'y', 5x + 6y + 10z - 30 = 0,$$

$$O'x'z' : , 9x + 11y + 19z - 57 = 0,$$

$$O'y'z' : 3x + 4y + 7z - 21 = 0,$$

$$5) \mathcal{P}' : -22x' + 10y' - 4z' - 7 = 0,$$

$$6) \ell' : \frac{x' - 48}{-89} = \frac{y' - 137}{-247} = \frac{z' - 73}{-133}.$$

$$6. \begin{cases} x = 5x' - 6y' + 14z' + 3, \\ y = -4x' + 5y' - 12z', \\ z = -3x' + 3y' - 7z', \end{cases}$$

$$\mathcal{P} : x - 4y + 5z - 2 = 0,$$

$$\ell: \frac{x-5}{-3} = \frac{y-4}{3} = \frac{z-7}{8}.$$

Ответ: 1) $O'[x = 3, y = 0, z = 0]$,

2) $O'x', \frac{x-3}{5} = \frac{y}{-4} = \frac{z}{-3}$,

$O'y', \frac{x-3}{-6} = \frac{y}{5} = \frac{z}{3}$,

$O'z', \frac{x-3}{14} = \frac{y}{-12} = \frac{z}{-7}$,

3) $Ox: \frac{x'-3}{-1} = \frac{y'-24}{-8} = \frac{z'-9}{-3}$,

$Oy: \frac{x'-3}{0} = \frac{y'-24}{-7} = \frac{z'-9}{-3}$,

$Oz: \frac{x'-3}{-2} = \frac{y'-24}{-4} = \frac{z'-9}{-1}$,

4) $O'x'y', -3x - 3y - z + 9 = 0$,

$O'x'z': -8x - 7y - 4z + 24 = 0$,

$O'y'z': -x - 2z + 3 = 0$,

5) $\mathcal{P}: 6x' - 11y' + 27z' + 1 = 0$,

6) $\ell': \frac{x'+16}{13} = \frac{y'+72}{29} = \frac{z'+25}{8}$.

7.
$$\begin{cases} x = -3x' + 7y' + 2z' - 1, \\ y = -3x' + 9y' + z' - 1, \\ z = x' - 2y' - z' - 1, \end{cases}$$

$$\mathcal{P}: 3x + 4y - 4z - 2 = 0,$$

$$\ell: \frac{x+2}{-5} = \frac{y-3}{-5} = \frac{z+6}{2}.$$

Ответ: 1) $O'[x = -1, y = -1, z = -1]$,

2) $O'x', \frac{x+1}{-3} = \frac{y+1}{-3} = \frac{z+1}{1}$,

$O'y', \frac{x+1}{7} = \frac{y+1}{9} = \frac{z+1}{-2}$,

$O'z', \frac{x+1}{2} = \frac{y+1}{1} = \frac{z+1}{-1}$,

3) $Ox: \frac{x'+15}{-7} = \frac{y'+4}{-2} = \frac{z'+8}{-3}$,

$Oy: \frac{x'+15}{3} = \frac{y'+4}{1} = \frac{z'+8}{1}$,

$Oz: \frac{x'+15}{-11} = \frac{y'+4}{-3} = \frac{z'+8}{-6}$,

4) $O'x'y', -3x + y - 6z - 8 = 0$,

$O'x'z': -2x + y - 3z - 4 = 0$,

$O'y'z': -7x + 3y - 11z - 15 = 0$,

5) $\mathcal{P}: -25x' + 65y' + 14z' - 5 = 0$,

6) $\ell': \frac{x'-74}{2} = \frac{y'-21}{1} = \frac{z'-37}{2}$.

8.
$$\begin{cases} x = 5x' + 10y' + 4z' + 2, \\ y = -3x' - 5y' - 2z' + 3, \\ z = x' + 3y' + z' + 3, \end{cases}$$

$$\mathcal{P}: 5x - 2y - 4z + 3 = 0,$$

$$\ell: \frac{x+3}{-3} = \frac{y-10}{4} = \frac{z+3}{1}.$$

Ответ: 1) $O'[x = 2, y = 3, z = 3]$,

2) $O'x', \frac{x-2}{5} = \frac{y-3}{-3} = \frac{z-3}{1}$,

$O'y', \frac{x-2}{10} = \frac{y-3}{-5} = \frac{z-3}{3}$,

$O'z', \frac{x-2}{4} = \frac{y-3}{-2} = \frac{z-3}{1}$,

3) $Ox: \frac{x'-8}{-1} = \frac{y'+1}{-1} = \frac{z'+8}{4}$,

$Oy: \frac{x'-8}{-2} = \frac{y'+1}{-1} = \frac{z'+8}{5}$,

$$Oz : , \frac{x' - 8}{0} = \frac{y' + 1}{2} = \frac{z' + 8}{-5},$$

$$4) O'x'y', 4x + 5y - 5z - 8 = 0,$$

$$O'x'z' : , -x - y + 2z - 1 = 0,$$

$$O'y'z' : -x - 2y + 8 = 0,$$

$$5) \mathcal{P}' : 27x' + 48y' + 20z' - 5 = 0,$$

$$6) \ell' : \frac{x' + 9}{5} = \frac{y' + 14}{-1} = \frac{z' - 45}{-3}.$$

$$9. \begin{cases} x = 4x' + 8y' - 11z' + 2, \\ y = -2x' - 3y' + 4z' + 2, \\ z = -x' - 2y' + 3z' - 1, \end{cases}$$

$$\mathcal{P} : 5x + y + 5z - 3 = 0,$$

$$\ell : \frac{x+5}{2} = \frac{y+7}{-8} = \frac{z+9}{0}.$$

$$\text{Ответ: 1) } O'[x = 2, y = 2, z = -1],$$

$$2) , O'x', \frac{x-2}{4} = \frac{y-2}{-2} = \frac{z+1}{-1},$$

$$O'y', \frac{x-2}{8} = \frac{y-2}{-3} = \frac{z+1}{-2},$$

$$O'z', \frac{x-2}{-11} = \frac{y-2}{4} = \frac{z+1}{3},$$

$$3) Ox : \frac{x'-5}{-1} = \frac{y'}{2} = \frac{z'-2}{1},$$

$$Oy : , \frac{x'-5}{-2} = \frac{y'}{1} = \frac{z'-2}{0},$$

$$Oz : , \frac{x'-5}{-1} = \frac{y'}{6} = \frac{z'-2}{4},$$

$$4) O'x'y', x + 4z + 2 = 0,$$

$$O'x'z' : , 2x + y + 6z = 0,$$

$$O'y'z' : -x - 2y - z + 5 = 0,$$

$$5) \mathcal{P}' : 13x' + 27y' - 36z' + 4 = 0,$$

$$6) \ell' : \frac{x' - 33}{-14} = \frac{y' + 71}{4} = \frac{z' + 39}{-2}.$$

$$10. \begin{cases} x = 5x' - y' - 6z' + 1, \\ y = 3x' - y' - 3z' - 2, \\ z = -4x' + y' + 5z' + 2, \end{cases}$$

$$\mathcal{P} : 3x + 2y - 5z + 4 = 0,$$

$$\ell : \frac{x-3}{1} = \frac{y}{7} = \frac{z-7}{5}.$$

$$\text{Ответ: 1) } O'[x = 1, y = -2, z = 2],$$

$$2) , O'x', \frac{x-1}{5} = \frac{y+2}{3} = \frac{z-2}{-4},$$

$$O'y', \frac{x-1}{-1} = \frac{y+2}{-1} = \frac{z-2}{1},$$

$$O'z', \frac{x-1}{-6} = \frac{y+2}{-3} = \frac{z-2}{5},$$

$$3) Ox : \frac{x'+6}{2} = \frac{y'+11}{3} = \frac{z'+3}{1},$$

$$Oy : , \frac{x'+6}{1} = \frac{y'+11}{-1} = \frac{z'+3}{1},$$

$$Oz : , \frac{x'+6}{3} = \frac{y'+11}{3} = \frac{z'+3}{2},$$

$$4) O'x'y', x + y + 2z - 3 = 0,$$

$$O'x'z' : , 3x - y + 3z - 11 = 0,$$

$$O'y'z' : 2x + y + 3z - 6 = 0,$$

$$5) \mathcal{P}' : 41x' - 10y' - 49z' - 7 = 0,$$

$$6) \ell' : \frac{x' - 21}{-24} = \frac{y' - 19}{-11} = \frac{z' - 14}{-18}.$$

$$11. \begin{cases} x = -2x' - y' - 4z' - 1, \\ y = x' + y' + 2z' - 1, \\ z = -3x' - y' - 7z' - 1, \end{cases}$$

$$\mathcal{P} : 2x - 5y - 3z + 4 = 0,$$

$$\ell : \frac{x-2}{-6} = \frac{y-8}{4} = \frac{z+6}{3}.$$

$$\text{Ответ: 1) } O'[x = -1, y = -1, z = -1],$$

$$2) \quad , O'x', \frac{x+1}{-2} = \frac{y+1}{1} = \frac{z+1}{-3},$$

$$O'y', \frac{x+1}{-1} = \frac{y+1}{1} = \frac{z+1}{-1},$$

$$O'z', \frac{x+1}{-4} = \frac{y+1}{2} = \frac{z+1}{-7},$$

$$3) \quad O'x : \frac{x'+6}{-5} = \frac{y'-3}{1} = \frac{z'-2}{2},$$

$$O'y : , \frac{x'+6}{-3} = \frac{y'-3}{2} = \frac{z'-2}{1},$$

$$O'z : , \frac{x'+6}{2} = \frac{y'-3}{0} = \frac{z'-2}{-1},$$

$$4) \quad O'x'y', 2x + y - z + 2 = 0,$$

$$O'x'z' : , x + 2y + 3 = 0,$$

$$O'y'z' : -5x - 3y + 2z - 6 = 0,$$

$$5) \quad \mathcal{P}' : -4y' + 3z' + 10 = 0,$$

$$6) \quad \ell' : \frac{x'+52}{-24} = \frac{y'-21}{-2} = \frac{z'-20}{11}.$$

$$12. \quad \begin{cases} x = 2x' + y' - 3z' + 3, \\ y = x' + y' - z', \\ z = 3x' + 2y' - 3z' - 3, \end{cases}$$

$$\mathcal{P} : 5x + 4y - 3z - 5 = 0,$$

$$\ell : \frac{x-4}{-4} = \frac{y-4}{-8} = \frac{z+3}{-7}.$$

$$\text{Ответ: } 1) \quad O'[x = 3, y = 0, z = -3],$$

$$2) \quad , O'x', \frac{x-3}{2} = \frac{y}{1} = \frac{z+3}{3},$$

$$O'y', \frac{x-3}{1} = \frac{y}{1} = \frac{z+3}{2},$$

$$O'z', \frac{x-3}{-3} = \frac{y}{-1} = \frac{z+3}{-3},$$

$$3) \quad O'x : \frac{x'-9}{-1} = \frac{y'+3}{0} = \frac{z'-6}{-1},$$

$$O'y : , \frac{x'-9}{-3} = \frac{y'+3}{3} = \frac{z'-6}{-1},$$

$$Oz : , \frac{x'-9}{2} = \frac{y'+3}{-1} = \frac{z'-6}{1},$$

$$4) \quad O'x'y', -x - y + z + 6 = 0,$$

$$O'x'z' : , 3y - z - 3 = 0,$$

$$O'y'z' : -x - 3y + 2z + 9 = 0,$$

$$5) \quad \mathcal{P}' : 5x' + 3y' - 10z' + 19 = 0,$$

$$6) \quad \ell' : \frac{x'+13}{-14} = \frac{y'-12}{17} = \frac{z'+5}{-5}.$$

$$13. \quad \begin{cases} x = 2x' + y' - z' - 3, \\ y = x' + y' + z' + 1, \\ z = 3x' + 2y' + z' - 1, \end{cases}$$

$$\mathcal{P} : 2x - 2y - z - 3 = 0,$$

$$\ell : \frac{x+1}{7} = \frac{y-1}{-10} = \frac{z+7}{2}.$$

$$\text{Ответ: } 1) \quad O'[x = -3, y = 1, z = -1],$$

$$2) \quad , O'x', \frac{x+3}{2} = \frac{y-1}{1} = \frac{z+1}{3},$$

$$O'y', \frac{x+3}{1} = \frac{y-1}{1} = \frac{z+1}{2},$$

$$O'z', \frac{x+3}{-1} = \frac{y-1}{1} = \frac{z+1}{1},$$

$$3) \quad O'x : \frac{x'-2}{-1} = \frac{y'+2}{2} = \frac{1+z'}{-1},$$

$$O'y : , \frac{x'-2}{-3} = \frac{y'+2}{5} = \frac{1+z'}{-1},$$

$$O'z : , \frac{x'-2}{2} = \frac{y'+2}{-3} = \frac{1+z'}{1},$$

$$4) \quad O'x'y', -x - y + z - 1 = 0,$$

$$O'x'z' : , 2x + 5y - 3z - 2 = 0,$$

$$O'y'z' : -x - 3y + 2z + 2 = 0,$$

$$5) \quad \mathcal{P}' : -x' - 2y' - 5z' - 10 = 0,$$

$$6) \quad \ell' : \frac{x'+14}{-27} = \frac{y'-22}{42} =$$

$$= \frac{z' + 8}{-5}.$$

$$14. \begin{cases} x = -5x' - 2y' + z' + 2, \\ y = -3x' - y' + z' + 1, \\ z = -6x' - 2y' + z' - 2, \end{cases}$$

$$\mathcal{P} : x + 4y + 4z - 2 = 0,$$

$$\ell : \frac{x-10}{-10} = \frac{y-4}{1} = \frac{z+5}{-7}.$$

Ответ: 1) $O'[x = 2, y = 1, z = -2]$,

2) $O'x', \frac{x-2}{-5} = \frac{y-1}{-3} = \frac{z+2}{-6},$

$O'y', \frac{x-2}{-2} = \frac{y-1}{-1} = \frac{z+2}{-2},$

$O'z', \frac{x-2}{1} = \frac{y-1}{1} = \frac{z+2}{1},$

3) $Ox : \frac{x'+4}{1} = \frac{y'-9}{-3} = \frac{z'+4}{0},$

$Oy : \frac{x'+4}{0} = \frac{y'-9}{1} = \frac{z'+4}{2},$

$Oz : \frac{x'+4}{-1} = \frac{y'-9}{2} = \frac{z'+4}{-1},$

4) $O'x'y', 2y - z - 4 = 0,$

$O'x'z' : -3x + y + 2z + 9 = 0,$

$O'y'z' : x - z - 4 = 0,$

5) $\mathcal{P}' : -41x' - 14y' + 9z' - 4 = 0,$

6) $\ell' : \frac{x'-11}{3} = \frac{y'+27}{-17} = \frac{z'-9}{-9}.$

$$15. \begin{cases} x = -12x' - 9y' + 8z' + 3, \\ y = x' + y' - 2z' - 2, \\ z = 4x' + 3y' - 3z' - 3, \end{cases}$$

$$\mathcal{P} : x + 2y + z + 3 = 0,$$

$$\ell : \frac{x}{-10} = \frac{y+1}{5} = \frac{z+6}{-2}.$$

Ответ: 1) $O'[x = 3, y = -2, z = -3]$,

2) $O'x', \frac{x-3}{-12} = \frac{y+2}{1} = \frac{z+3}{4},$

$O'y', \frac{x-3}{-9} = \frac{y+2}{1} = \frac{z+3}{3},$

$O'z', \frac{x-3}{8} = \frac{y+2}{-2} = \frac{z+3}{-3},$

3) $Ox : \frac{x'-15}{3} = \frac{y'+25}{-5} = \frac{z'+6}{-1},$

$Oy : \frac{x'-15}{-3} = \frac{y'+25}{4} = \frac{z'+6}{0},$

$Oz : \frac{x'-15}{10} = \frac{y'+25}{-16} = \frac{z'+6}{-3},$

4) $O'x'y', -x - 3z - 6 = 0,$

$O'x'z' : -5x + 4y - 16z - 25 = 0,$

$O'y'z' : 3x - 3y + 10z + 15 = 0,$

5) $\mathcal{P}' : -6x' - 4y' + z' - 1 = 0,$

6) $\ell' : \frac{x'+42}{65} = \frac{y'-67}{-102} = \frac{z'-12}{-16}.$

$$16. \begin{cases} x = 5x' - 2y' + 6z' + 2, \\ y = 3x' - y' + 4z' + 2, \\ z = 8x' - 3y' + 9z' - 1, \end{cases}$$

$\mathcal{P} : 4x - 2y + 4z + 5 = 0,$

$\ell : \frac{x+2}{5} = \frac{y+8}{-6} = \frac{z+1}{4}.$

Ответ: 1) $O'[x = 2, y = 2, z = -1]$,

2) $O'x', \frac{x-2}{5} = \frac{y-2}{3} =$

$$= \frac{z+1}{8},$$

$$O'y', \frac{x-2}{-2} = \frac{y-2}{-1} = \frac{z+1}{-3},$$

$$O'z', \frac{x-2}{6} = \frac{y-2}{4} = \frac{z+1}{9},$$

$$3) O_x : \frac{x'-8}{-3} = \frac{y'-6}{-5} =$$

$$= \frac{z'+5}{1},$$

$$O_y : \frac{x'-8}{0} = \frac{y'-6}{3} = \frac{z'+5}{1},$$

$$O_z : \frac{x'-8}{2} = \frac{y'-6}{2} = \frac{z'+5}{-1},$$

$$4) O'x'y', x+y-z-5=0,$$

$$O'x'z' : -5x+3y+2z+6=$$

$$= 0,$$

$$O'y'z' : -3x+2z+8=0,$$

$$5) \mathcal{P}' : 46x' - 18y' + 52z' +$$

$$+ 5 = 0,$$

$$6) \ell' : \frac{x'-12}{7} = \frac{y'+10}{35} =$$

$$= \frac{z'+14}{5}.$$

$$17. \begin{cases} x = -2x' + 3y' - z' - 1, \\ y = -x' - y' + z' + 1, \\ z = -x' + 6y' - 3z' - 1, \end{cases}$$

$$\mathcal{P} : x + 2y + 4z - 4 = 0,$$

$$\ell : \frac{x+7}{8} = \frac{y-5}{-7} = \frac{z-10}{-4}.$$

Ответ: 1) $O'[x = -1, y =$
 $= 1, z = -1],$
 2) $O'x', \frac{x+1}{-2} = \frac{y-1}{-1} =$
 $= \frac{z+1}{-1},$
 $O'y', \frac{x+1}{3} = \frac{y-1}{-1} = \frac{z+1}{6},$
 $O'z', \frac{x+1}{-1} = \frac{y-1}{1} = \frac{z+1}{-3},$
 3) $O_x : \frac{x'+4}{-3} = \frac{y'+6}{-4} =$
 $= \frac{z'+11}{-7},$

$$O_y : \frac{x'+4}{3} = \frac{y'+6}{5} = \frac{z'+11}{9},$$

$$O_z : \frac{x'+4}{2} = \frac{y'+6}{3} = \frac{z'+11}{5},$$

$$4) O'x'y', -7x+9y+5z-11=$$

$$= 0,$$

$$O'x'z' : -4x+5y+3z-6=$$

$$= 0,$$

$$O'y'z' : -3x+3y+2z-4=$$

$$= 0,$$

$$5) \mathcal{P}' : -8x' + 25y' - 11z' -$$

$$- 7 = 0,$$

$$6) \ell' : \frac{x'-52}{53} = \frac{y'-77}{79} =$$

$$= \frac{z'-133}{139}.$$

$$18. \begin{cases} x = x' - y' + z', \\ y = -2x' + y' + z' + 3, \\ z = 2x' - y' - 2z' - 1, \end{cases}$$

$$\mathcal{P} : 5x + 5y - z + 3 = 0,$$

$$\ell : \frac{x+5}{3} = \frac{y}{2} = \frac{z+6}{-10}.$$

Ответ: 1) $O'[x = 0, y =$
 $= 3, z = -1],$
 2) $O'x', \frac{x}{1} = \frac{y-3}{-2} = \frac{z+1}{2},$
 $O'y', \frac{x}{-1} = \frac{y-3}{1} = \frac{z+1}{-1},$
 $O'z', \frac{x}{1} = \frac{y-3}{1} = \frac{z+1}{-2},$
 3) $O_x : \frac{x'-7}{-1} = \frac{y'-9}{-2} =$
 $= \frac{z'-2}{0},$
 $O_y : \frac{x'-7}{-3} = \frac{y'-9}{-4} = \frac{z'-2}{-1},$
 $O_z : \frac{x'-7}{-2} = \frac{y'-9}{-3} = \frac{z'-2}{-1},$
 4) $O'x'y', -y - z + 2 = 0,$
 $O'x'z' : -2x - 4y - 3z + 9 =$
 $= 0,$
 $O'y'z' : -x - 3y - 2z + 7 = 0,$

$$5) \mathcal{P}' : -7x' + y' + 12z' + 19 = 0,$$

$$6) \ell' : \frac{x' - 24}{-11} = \frac{y' - 37}{-16} = \frac{z' - 8}{-8}.$$

$$19. \begin{cases} x = x' + 11y' + 3z', \\ y = -x' - 12y' - 3z' + 3, \\ z = -x' - 18y' - 4z' - 1, \end{cases}$$

$$\mathcal{P} : 5x + y - z - 1 = 0,$$

$$\ell : \frac{x-7}{9} = \frac{y-4}{-4} = \frac{z+2}{10}.$$

$$\text{Ответ: 1) } O'[x = 0, y = 3, z = -1],$$

$$2) , O'x', \frac{x}{1} = \frac{y-3}{-1} = \frac{z+1}{-1},$$

$$O'y', \frac{x}{11} = \frac{y-3}{-12} = \frac{z+1}{-18},$$

$$O'z', \frac{x}{3} = \frac{y-3}{-3} = \frac{z+1}{-4},$$

$$3) Ox : \frac{x' - 33}{-6} = \frac{y' - 3}{-1} = \frac{z' + 22}{6},$$

$$Oy : , \frac{x' - 33}{-10} = \frac{y' - 3}{-1} = \frac{z' + 22}{7},$$

$$Oz : , \frac{x' - 33}{3} = \frac{y' - 3}{0} = \frac{z' + 22}{-1},$$

$$4) O'x'y', 6x + 7y - z - 22 = 0,$$

$$O'x'z' : , -x - y + 3 = 0,$$

$$O'y'z' : -6x - 10y + 3z + 33 = 0,$$

$$5) \mathcal{P}' : 5x' + 61y' + 16z' + 3 = 0,$$

$$6) \ell' : \frac{x' + 55}{-16} = \frac{y' + 8}{5} = \frac{z' - 50}{-16}.$$

$$20. \begin{cases} x = -x' - 2y' - 2z' + 3, \\ y = 3x' + 5y' + 3z' + 1, \\ z = 4x' + 6y' + 3z' - 3, \end{cases}$$

$$\mathcal{P} : 2x + 4y - 2z - 1 = 0,$$

$$\ell : \frac{x-8}{0} = \frac{y-4}{-9} = \frac{z-8}{2}.$$

$$\text{Ответ: 1) } O'[x = 3, y = 1, z = -3],$$

$$2) , O'x', \frac{x-3}{-1} = \frac{y-1}{3} = \frac{z+3}{4},$$

$$O'y', \frac{x-3}{-2} = \frac{y-1}{5} = \frac{z+3}{6},$$

$$O'z', \frac{x-3}{-2} = \frac{y-1}{3} = \frac{z+3}{3},$$

$$3) Ox : \frac{x' - 27}{-3} = \frac{y' + 23}{3} = \frac{z' - 11}{-2},$$

$$Oy : , \frac{x' - 27}{-6} = \frac{y' + 23}{5} = \frac{z' - 11}{-2},$$

$$Oz : , \frac{x' - 27}{4} = \frac{y' + 23}{-3} = \frac{z' - 11}{1},$$

$$4) O'x'y', -2x - 2y + z + 11 = 0,$$

$$O'x'z' : , 3x + 5y - 3z - 23 = 0,$$

$$O'y'z' : -3x - 6y + 4z + 27 = 0,$$

$$5) \mathcal{P}' : 2x' + 4y' + 2z' + 15 = 0,$$

$$6) \ell' : \frac{x' - 11}{-62} = \frac{y' + 3}{51} = \frac{z' + 5}{-20}.$$

$$21. \begin{cases} x = 3x' - y' + 3z' - 3, \\ y = -5x' + 2y' - 6z' + 1, \\ z = -x' + y' - 2z' + 3, \end{cases}$$

$$\mathcal{P} : 3x + 2y - 3z - 2 = 0,$$

$$\ell : \frac{x+2}{5} = \frac{y-2}{-5} = \frac{z+1}{-7}.$$

Ответ: 1) $O'[x = -3, y = 1, z = 3],$

2) $O'x', \frac{x+3}{3} = \frac{y-1}{-5} = \frac{z-3}{-1},$

$O'y', \frac{x+3}{-1} = \frac{y-1}{2} = \frac{z-3}{1},$

$O'z', \frac{x+3}{3} = \frac{y-1}{-6} = \frac{z-3}{-2},$

3) $Ox : \frac{x'-5}{2} = \frac{y'+18}{-4} = \frac{z'+10}{-3},$

$Oy : \frac{x'-5}{1} = \frac{y'+18}{-3} = \frac{z'+10}{-2},$

$Oz : \frac{x'-5}{0} = \frac{y'+18}{3} = \frac{z'+10}{1},$

4) $O'x'y', -3x - 2y + z - 10 = 0,$

$O'x'z', -4x - 3y + 3z - 18 = 0,$

$O'y'z', 2x + y + 5 = 0,$

5) $\mathcal{P}' : 2x' - 2y' + 3z' - 18 = 0,$

6) $\ell' : \frac{x'-3}{-5} = \frac{y'+19}{26} = \frac{z'+9}{12}.$

$$22. \begin{cases} x = 4x' + 5y' - 4z' - 2, \\ y = 16x' + 20y' - 17z' - 3, \\ z = 23x' + 29y' - 25z' - 1, \end{cases}$$

$$\mathcal{P} : 3x - 5y - 5z - 5 = 0,$$

$$\ell : \frac{x+4}{4} = \frac{y-5}{-8} = \frac{z+1}{1}.$$

Ответ: 1) $O'[x = -2, y = -3, z = -1],$

2) $O'x', \frac{x+2}{4} = \frac{y+3}{16} = \frac{z+1}{23},$

$O'y', \frac{x+2}{5} = \frac{y+3}{20} = \frac{z+1}{29},$

$O'z', \frac{x+2}{-4} = \frac{y+3}{-17} = \frac{z+1}{-25},$

3) $Ox : \frac{x'-8}{-7} = \frac{y'+2}{9} = \frac{z'-5}{4},$

$Oy : \frac{x'-8}{9} = \frac{y'+2}{-8} = \frac{z'-5}{-1},$

$Oz : \frac{x'-8}{-5} = \frac{y'+2}{4} = \frac{z'-5}{0},$

4) $O'x'y', 4x - y + 5 = 0,$

$O'x'z', 9x - 8y + 4z - 2 = 0,$

$O'y'z', -7x + 9y - 5z + 8 = 0,$

5) $\mathcal{P}' : -183x' - 230y' + 198z' + 9 = 0,$

6) $\ell' : \frac{x'-86}{105} = \frac{y'+82}{-104} = \frac{z'+16}{-24}.$

$$23. \begin{cases} x = -2x' + y' - 2z' + 1, \\ y = -2x' + 2y' - 3z', \\ z = 5x' - 4y' + 6z' - 2, \end{cases}$$

$$\mathcal{P} : x + y - z - 2 = 0,$$

$$\ell : \frac{x-6}{-7} = \frac{y-10}{7} = \frac{z-8}{2}.$$

Ответ: 1) $O'[x = 1, y = 0, z = -2],$

2) $O'x', \frac{x-1}{-2} = \frac{y}{-2} = \frac{2+z}{5},$

$O'y', \frac{x-1}{1} = \frac{y}{2} = \frac{2+z}{-4},$

$$O'z', \frac{x-1}{-2} = \frac{y}{-3} = \frac{2+z}{6},$$

$$3) Ox : \frac{x'-2}{0} = \frac{y'+1}{-3} = \frac{z'+2}{-2},$$

$$Oy : \frac{x'-2}{2} = \frac{y'+1}{-2} = \frac{z'+2}{-3},$$

$$Oz : \frac{x'-2}{1} = \frac{y'+1}{-2} = \frac{z'+2}{-2},$$

$$4) O'x'y', -2x - 3y - 2z - 2 = 0,$$

$$O'x'z' : -3x - 2y - 2z - 1 = 0,$$

$$O'y'z' : 2y + z + 2 = 0,$$

$$5) \mathcal{P}' : -9x' + 7y' - 11z' + 1 = 0,$$

$$6) \ell' : \frac{x'-30}{-16} = \frac{y'+55}{-3} = \frac{z'+60}{11}.$$

$$24. \begin{cases} x = -x' - 3y' - 6z' - 1, \\ y = 3x' + 7y' + 15z' - 3, \\ z = 3x' + 6y' + 13z', \end{cases}$$

$$\mathcal{P} : 5x - y + 4z + 2 = 0,$$

$$\ell : \frac{x+6}{-9} = \frac{y-7}{7} = \frac{z-7}{2}.$$

$$\text{Ответ: } 1) O'[x = -1, y = -3, z = 0],$$

$$2) , O'x', \frac{x+1}{-1} = \frac{y+3}{3} = \frac{z}{3},$$

$$O'y', \frac{x+1}{-3} = \frac{y+3}{7} = \frac{z}{6},$$

$$O'z', \frac{x+1}{-6} = \frac{y+3}{15} = \frac{z}{13},$$

$$3) Ox : \frac{x'+10}{-1} = \frac{y'+21}{-6} = \frac{z'-12}{3},$$

$$Oy : \frac{x'+10}{-3} = \frac{y'+21}{-5} = \frac{z'-12}{3},$$

$$Oz : \frac{x'+10}{3} = \frac{y'+21}{3} = \frac{z'-12}{-2},$$

$$4) O'x'y', 3x + 3y - 2z + 12 = 0,$$

$$O'x'z' : -6x - 5y + 3z - 21 = 0,$$

$$O'y'z' : -x - 3y + 3z - 10 = 0,$$

$$5) \mathcal{P}' : 4x' + 2y' + 7z' = 0,$$

$$6) \ell' : \frac{x'+4}{6} = \frac{y'-1}{-25} = \frac{z'-1}{10}.$$

$$25. \begin{cases} x = -x' - y' + z' - 2, \\ y = x' + 2y' - z' + 1, \\ z = x' + y' - 2z', \end{cases}$$

$$\mathcal{P} : 3x + 5y + 3z - 1 = 0,$$

$$\ell : \frac{x}{7} = \frac{y+3}{-5} = \frac{z-7}{-4}.$$

$$\text{Ответ: } 1) O'[x = -2, y = 1, z = 0],$$

$$2) , O'x', \frac{x+2}{-1} = \frac{y-1}{1} = \frac{z}{1},$$

$$O'y', \frac{x+2}{-1} = \frac{y-1}{2} = \frac{z}{1},$$

$$O'z', \frac{x+2}{1} = \frac{y-1}{-1} = \frac{z}{-2},$$

$$3) Ox : \frac{x'+5}{-3} = \frac{y'-1}{1} = \frac{z'+2}{-1},$$

$$Oy : \frac{x'+5}{-1} = \frac{y'-1}{1} = \frac{z'+2}{0},$$

$$Oz : \frac{x'+5}{-1} = \frac{y'-1}{0} = \frac{z'+2}{-1},$$

$$4) O'x'y', -x - z - 2 = 0,$$

$$O'x'z' : x + y + 1 = 0,$$

$$O'y'z' : -3x - y - z - 5 = 0,$$

$$5) \mathcal{P}' : 5x' + 10y' - 8z' - 2 = 0,$$

$$6) \ell' : \frac{x'+9}{12} = \frac{y'+2}{-2} = \frac{z'+9}{3}.$$

$$26. \begin{cases} x = -12x' - 6y' - 5z' - 3, \\ y = 10x' + 5y' + 4z', \\ z = -9x' - 5y' - 6z', \end{cases}$$

$$\mathcal{P} : 5x + 2y - z + 3 = 0,$$

$$\ell : \frac{x-9}{1} = \frac{y+5}{6} = \frac{z-1}{2}.$$

Ответ: 1) $O'[x = -3, y = 0, z = 0],$

2) $O'x', \frac{x+3}{-12} = \frac{y}{10} = \frac{z}{-9},$

$O'y', \frac{x+3}{-6} = \frac{y}{5} = \frac{z}{-5},$

$O'z', \frac{x+3}{-5} = \frac{y}{4} = \frac{z}{-6},$

3) $Ox : \frac{x'+30}{-10} = \frac{y'-72}{24} = \frac{z'+15}{-5},$

$Oy : \frac{x'+30}{-11} = \frac{y'-72}{27} = \frac{z'+15}{-6},$

$Oz : \frac{x'+30}{1} = \frac{y'-72}{-2} = \frac{z'+15}{0},$

4) $O'x'y', -5x - 6y - 15 = 0,$

$O'x'z' : , 24x + 27y - 2z + 72 = 0,$

$O'y'z' : -10x - 11y + z - 30 = 0,$

5) $\mathcal{P}' : -31x' - 15y' - 11z' - 12 = 0,$

6) $\ell' : \frac{x'+64}{74} = \frac{y'-151}{-182} = \frac{z'+30}{41}.$

$$27. \begin{cases} x = 5x' - 3y' - 5z', \\ y = 13x' - 10y' - 15z' - 3, \\ z = -5x' + 4y' + 6z' - 2, \end{cases}$$

$$\mathcal{P} : 3x - 3y + z - 4 = 0,$$

$$\ell : \frac{x-6}{-1} = \frac{y-1}{-4} = \frac{z-3}{5}.$$

Ответ: 1) $O'[x = 0, y = -3, z = -2],$

2) $O'x', \frac{x}{5} = \frac{y+3}{13} = \frac{z+2}{-5},$

$O'y', \frac{x}{-3} = \frac{y+3}{-10} = \frac{z+2}{4},$

$O'z', \frac{x}{-5} = \frac{y+3}{-15} = \frac{z+2}{6},$

3) $Ox : \frac{x'-16}{0} = \frac{y'+35}{3} = \frac{z'-37}{-2},$

$Oy : , \frac{x'-16}{2} = \frac{y'+35}{-5} = \frac{z'-37}{5},$

$Oz : , \frac{x'-16}{5} = \frac{y'+35}{-10} = \frac{z'-37}{11},$

4) $O'x'y', -2x + 5y + 11z + 37 = 0,$

$O'x'z' : , 3x - 5y - 10z - 35 = 0,$

$O'y'z' : 2y + 5z + 16 = 0,$

5) $\mathcal{P}' : -29x' + 25y' + 36z' + 3 = 0,$

6) $\ell' : \frac{x'-33}{-17} = \frac{y'+52}{33} = \frac{z'-63}{-37}.$

$$28. \begin{cases} x = -4x' + 2y' + z' + 1, \\ y = -5x' + 2y' + 2z' - 2, \\ z = 6x' - 3y' - 2z' + 3, \end{cases}$$

$$\mathcal{P} : 5x + 2y + z - 5 = 0,$$

$$\ell : \frac{x+7}{5} = \frac{y+3}{9} = \frac{z-9}{8}.$$

Ответ: 1) $O'[x = 1, y = -2, z = 3],$

2) $O'x', \frac{x-1}{-4} = \frac{y+2}{-5} =$

$$= \frac{z-3}{6},$$

$$O'y', \frac{x-1}{2} = \frac{y+2}{2} = \frac{z-3}{-3},$$

$$O'z', \frac{x-1}{1} = \frac{y+2}{2} = \frac{z-3}{-2},$$

$$3) O_x : \frac{x'-6}{-2} = \frac{y'-7}{-2} =$$

$$= \frac{z'-9}{-3},$$

$$O_y : \frac{x'-6}{-1} = \frac{y'-7}{-2} = \frac{z'-9}{0},$$

$$O_z : \frac{x'-6}{-2} = \frac{y'-7}{-3} = \frac{z'-9}{-2},$$

$$4) O'x'y', -3x - 2z + 9 = 0,$$

$$O'x'z' : -2x - 2y - 3z + 7 =$$

$$= 0,$$

$$O'y'z' : -2x - y - 2z + 6 = 0,$$

$$5) \mathcal{P}' : -24x' + 11y' + 7z' -$$

$$-1 = 0,$$

$$6) \ell' : \frac{x'-5}{35} = \frac{y'}{52} = \frac{z'-12}{31}.$$

$$29. \begin{cases} x = 14x' - 5y' - 3z' - 2, \\ y = -8x' + 3y' + 2z' + 1, \\ z = -7x' + 2y' + z' - 2, \end{cases}$$

$$\mathcal{P} : 2x - 3y + 4z + 3 = 0,$$

$$\ell : \frac{x+6}{-1} = \frac{y+2}{0} = \frac{z-10}{-2}.$$

Ответ: 1) $O'[x = -2, y =$
 $= 1, z = -2],$

$$2) , O'x', \frac{x+2}{14} = \frac{y-1}{-8} =$$

$$= \frac{z+2}{-7},$$

$$O'y', \frac{x+2}{-5} = \frac{y-1}{3} = \frac{z+2}{2},$$

$$O'z', \frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+2}{1},$$

$$3) O_x : \frac{x'+3}{-1} = \frac{y'+13}{-6} =$$

$$= \frac{z'-7}{5},$$

$$O_y : \frac{x'+3}{-1} = \frac{y'+13}{-7} = \frac{z'-7}{7},$$

$$O_z : \frac{x'+3}{-1} = \frac{y'+13}{-4} = \frac{z'-7}{2},$$

$$4) O'x'y', 5x + 7y + 2z + 7 =$$

$$= 0,$$

$$O'x'z' : -6x - 7y - 4z - 13 =$$

$$= 0,$$

$$O'y'z' : -x - y - z - 3 = 0,$$

$$5) \mathcal{P}' : 24x' - 11y' - 8z' -$$

$$-12 = 0,$$

$$6) \ell' : \frac{x'+5}{-3} = \frac{y'+3}{-14} =$$

$$= \frac{z'+17}{9}.$$

$$30. \begin{cases} x = -5x' - 3y' - 4z' + 3, \\ y = 23x' + 10y' + 23z' - 2, \\ z = 6x' + y' + 8z' + 3, \end{cases}$$

$$\mathcal{P} : 4x - 4y + 4z + 4 = 0,$$

$$\ell : \frac{x+2}{5} = \frac{y-7}{-5} = \frac{z-8}{7}.$$

Ответ: 1) $O'[x = 3, y = -$
 $-2, z = 3],$

$$2) , O'x', \frac{x-3}{-5} = \frac{y+2}{23} =$$

$$= \frac{z-3}{6},$$

$$O'y', \frac{x-3}{-3} = \frac{y+2}{10} = \frac{z-3}{1},$$

$$O'z', \frac{x-3}{-4} = \frac{y+2}{23} = \frac{z-3}{8},$$

$$3) O_x : \frac{x'+44}{57} = \frac{y'-37}{-46} =$$

$$= \frac{z'-28}{-37},$$

$$O_y : \frac{x'+44}{20} = \frac{y'-37}{-16} =$$

$$= \frac{z'-28}{-13},$$

$$O_z : \frac{x'+44}{-29} = \frac{y'-37}{23} =$$

$$= \frac{z'-28}{19},$$

$$4) O'x'y', -37x - 13y + 19z +$$

$$+ 28 = 0,$$

$$O'x'z' : , -46x - 16y + 23z +$$

$$+ 37 = 0,$$

$$O'y'z' : 57x + 20y - 29z -$$

$$- 44 = 0,$$

$$5) \mathcal{P}' : -88x' - 48y' - 76z' +$$

$$+ 36 = 0,$$

$$6) \ell' : \frac{x' + 250}{18} = \frac{y' - 201}{-11} =$$

$$= \frac{z' - 163}{-13}.$$

Задача 1.12. За начало новой системы координат $O'x'y'z'w'$ принята точка O' , а за единичные векторы ее осей — \vec{e}'_1 , \vec{e}'_2 , \vec{e}'_3 и \vec{e}'_4 . Координаты заданы в старой системе координат. Найти выражение новых координат x', y', z', w' через старые x, y, z, w .

$$1. O'(2, -1, 3, -3), \vec{e}'_1 = (-7, -9, 2, -8), \vec{e}'_2 = (-7, -14, -1, -10),$$

$$\vec{e}'_3 = (2, 7, 2, 4), \vec{e}'_4 = (2, 5, 1, 3).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 3x - 4y + 5z + 3w - 16 \\ -4x + 6y - 7z - 5w + 20 \\ -7x + 7y - 9z - 4w + 36 \\ 4x + 2z - 3w - 23 \end{pmatrix}$$

$$2. O'(-3, -2, 1, 3), \vec{e}'_1 = (4, -4, -3, -9), \vec{e}'_2 = (-4, 3, 2, 8),$$

$$\vec{e}'_3 = (-2, 3, 2, 5), \vec{e}'_4 = (-3, 4, 2, 7).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -6x + 2y + z - 4w - 3 \\ -5x + y + z - 3w - 5 \\ -7x + 2y + 3z - 5w - 5 \\ 3x - 2z + 2w + 5 \end{pmatrix}$$

$$3. O'(3, 0, 2, 0), \vec{e}'_1 = (3, -1, 1, -3), \vec{e}'_2 = (-6, 1, -6, 11),$$

$$\vec{e}'_3 = (-8, 3, -7, 15), \vec{e}'_4 = (3, -1, 3, -6).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x - 3y + 4z + 3w - 11 \\ -3y + 3z + 2w - 6 \\ x - 6y + 9z + 6w - 21 \\ 2x - 19y + 26z + 17w - 58 \end{pmatrix}$$

$$4. O'(0, -3, -3, -3), \vec{e}'_1 = (-8, -1, 9, 4), \vec{e}'_2 = (-7, 2, 7, 4),$$

$$\vec{e}'_3 = (-16, -1, 18, 8), \vec{e}'_4 = (-9, 3, 9, 5).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -13x - 2y - 9z - 6w - 51 \\ 9x + 4z + 9w + 39 \\ 7x + y + 5z + 3w + 27 \\ -8x - 4z - 7w - 33 \end{pmatrix}$$

5. $O'(3, -3, 0, 2)$, $\vec{e}'_1 = (-5, -3, 2, 8)$, $\vec{e}'_2 = (1, 1, -2, -5)$,
 $\vec{e}'_3 = (-1, -1, 1, 3)$, $\vec{e}'_4 = (12, 7, -5, -20)$.

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -3x - 2y + 10z - 5w + 13 \\ x - y - 3z + w - 8 \\ 3x - 3y - 5z + 2w - 22 \\ -x - y + 4z - 2w + 4 \end{pmatrix}$$

6. $O'(-1, 2, -2, -3)$, $\vec{e}'_1 = (-3, 2, -2, -5)$, $\vec{e}'_2 = (-3, -5, -3, -2)$,
 $\vec{e}'_3 = (-3, -1, -3, -5)$, $\vec{e}'_4 = (2, 1, 2, 3)$.

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -x + z + 1 \\ -x - y + 3z - w + 4 \\ -x - 5y + 14z - 7w + 16 \\ -4x - 9y + 27z - 12w + 32 \end{pmatrix}$$

7. $O'(2, -2, 0, 2)$, $\vec{e}'_1 = (-5, 3, 4, -2)$, $\vec{e}'_2 = (-2, 1, 2, -1)$,
 $\vec{e}'_3 = (2, -2, -2, 1)$, $\vec{e}'_4 = (5, -3, -5, 3)$.

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -x - z + 2 \\ x - y + 4z + 4w - 12 \\ -x - y - z - w + 2 \\ z + 2w - 4 \end{pmatrix}$$

8. $O'(0, 0, -1, -1)$, $\vec{e}'_1 = (-2, -4, 5, 19)$, $\vec{e}'_2 = (2, 4, -4, -15)$,
 $\vec{e}'_3 = (-1, -6, 4, 17)$, $\vec{e}'_4 = (-1, -1, 3, 11)$.

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -14x + 5y - 47z + 12w - 35 \\ -7x + 3y - 27z + 7w - 20 \\ 3x - y + 8z - 2w + 6 \\ 10x - 3y + 32z - 8w + 24 \end{pmatrix}$$

9. $O'(-2, -2, 2, 0)$, $\vec{e}'_1 = (14, -14, -23, 4)$, $\vec{e}'_2 = (-39, 37, 60, -13)$,
 $\vec{e}'_3 = (25, -24, -39, 8)$, $\vec{e}'_4 = (15, -14, -23, 5)$.

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -5x + y - 3z + 4w - 2 \\ 6x + y + 2z - 6w + 10 \\ 11x - 2y + 6z - 11w + 6 \\ 2x + 5y - 2z - w + 18 \end{pmatrix}$$

10. $O'(1, 2, 0, -3)$, $\vec{e}'_1 = (3, -1, 4, 2)$, $\vec{e}'_2 = (-1, 1, 1, -1)$,
 $\vec{e}'_3 = (5, -3, 3, 4)$, $\vec{e}'_4 = (-3, 2, -1, -3)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 3x + 4y - z - 11 \\ -5x - 5y + 2z + w + 18 \\ -2x - 4y + z - w + 7 \\ x - y - 2w - 5 \end{pmatrix}$$

11. $O'(1, -2, -3, -1)$, $\vec{e}'_1 = (-4, -7, -1, -5)$, $\vec{e}'_2 = (-4, -7, -1, -4)$,
 $\vec{e}'_3 = (-11, -19, -2, -12)$, $\vec{e}'_4 = (14, 24, 1, 17)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 3x - y - z - w - 9 \\ 26x - 16y + 3z + w - 48 \\ -17x + 10y - 2z + 31 \\ -5x + 3y - z + 8 \end{pmatrix}$$

12. $O'(1, -3, -3, 2)$, $\vec{e}'_1 = (3, -5, 9, -3)$, $\vec{e}'_2 = (-1, 2, -3, 1)$,
 $\vec{e}'_3 = (4, -8, 13, -7)$, $\vec{e}'_4 = (3, -7, 10, -7)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 10x + y - 3z - w - 14 \\ 9x + 3y - 2z - 2w - 2 \\ -11x + 4z + w + 21 \\ 8x - 3z - w - 15 \end{pmatrix}$$

13. $O'(-1, 3, -3, -3)$, $\vec{e}'_1 = (-2, 3, 3, -1)$, $\vec{e}'_2 = (-6, 5, 6, -4)$,
 $\vec{e}'_3 = (5, -4, -4, 3)$, $\vec{e}'_4 = (3, -2, -2, 2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 2x + 2y - z - 2w - 13 \\ -y + z + 6 \\ 4x + 2y - z - 5w - 20 \\ -5x - 4y + 3z + 7w + 37 \end{pmatrix}$$

14. $O'(1, 2, 2, -2)$, $\vec{e}'_1 = (-10, -5, -5, 8)$, $\vec{e}'_2 = (-46, -21, -18, 30)$,
 $\vec{e}'_3 = (-32, -15, -13, 21)$, $\vec{e}'_4 = (19, 9, 8, -13)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -3x + 10y - 9z - 3w - 5 \\ -x + 3y - z - 3 \\ -3x + 8y - 10z - 5w - 3 \\ -9x + 26y - 24z - 10w - 15 \end{pmatrix}$$

15. $O'(-3, -2, 0, -1)$, $\vec{e}'_1 = (5, -3, -2, 12)$, $\vec{e}'_2 = (-1, 1, 1, -3)$,
 $\vec{e}'_3 = (2, -1, -1, 4)$, $\vec{e}'_4 = (-1, -1, 1, 2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -2x + 5y - z + 2w + 6 \\ 2x - 2y + 2z - w + 1 \\ 7x - 15y + 4z - 6w - 15 \\ x - 3y + z - w - 4 \end{pmatrix}$$

$$16. O'(-3, -1, 2, -3), \vec{e}'_1 = (5, 8, 6, -4), \vec{e}'_2 = (2, 5, 3, -2), \\ \vec{e}'_3 = (-6, -11, -8, 5), \vec{e}'_4 = (3, 6, 4, -3).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 3x + y - 3z + w + 19 \\ 2x + 2y - 3z + 2w + 20 \\ 2x + y - 3z + 13 \\ -2x - y + z - 3w - 18 \end{pmatrix}$$

$$17. O'(-2, 2, 0, 1), \vec{e}'_1 = (-12, -8, -3, -19), \vec{e}'_2 = (-3, -2, -1, -5), \\ \vec{e}'_3 = (1, 1, 1, 2), \vec{e}'_4 = (-14, -9, -4, -22).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x - 2y + z + 6 \\ 5x + 4y + z - 5w + 7 \\ 2y + z - w - 3 \\ -2x + y - z + w - 7 \end{pmatrix}$$

$$18. O'(-2, 0, -3, 2), \vec{e}'_1 = (3, -1, 3, 3), \vec{e}'_2 = (-8, 3, -4, -7), \\ \vec{e}'_3 = (-11, 4, -5, -9), \vec{e}'_4 = (21, -8, 10, 18).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -y + z - w + 5 \\ -12y + 3z - 7w + 23 \\ -2x - 3y + w - 6 \\ -x - 6y + z - 2w + 5 \end{pmatrix}$$

$$19. O'(2, 3, 0, 3), \vec{e}'_1 = (3, -17, -12, -11), \vec{e}'_2 = (1, -3, -2, -2), \\ \vec{e}'_3 = (12, -46, -31, -28), \vec{e}'_4 = (-8, 31, 21, 19).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -x - y + 2z - w + 8 \\ -y + 6z - 5w + 18 \\ -3x - 7y + 11z - 2w + 33 \\ -5x - 11y + 18z - 4w + 55 \end{pmatrix}$$

$$20. O'(-3, 1, -2, 1), \vec{e}'_1 = (-17, 38, 19, -25), \vec{e}'_2 = (-6, 14, 7, -9), \\ \vec{e}'_3 = (-3, 7, 4, -5), \vec{e}'_4 = (-6, 13, 7, -9).$$

$$\text{Ответ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} x - 3z - 3w \\ -x + y + 5z + 6w \\ 4x - 3z - 5w + 11 \\ -4x - y + 5z + 5w - 6 \end{pmatrix}$$

$$21. O'(2, 0, -1, 2), \vec{e}'_1 = (-1, 2, -2, -2), \vec{e}'_2 = (1, -2, 4, 3), \\ \vec{e}'_3 = (2, -3, 3, 2), \vec{e}'_4 = (-2, 1, 2, 3).$$

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -7x - 5y + 3z - 5w + 27 \\ 6x + 5y - z + 3w - 19 \\ -10x - 8y + 3z - 6w + 35 \\ -4x - 3y + z - 2w + 13 \end{pmatrix}$$

22. $O'(-2, 2, 0, -2)$, $\vec{e}'_1 = (5, 11, 7, 8)$, $\vec{e}'_2 = (-5, -9, -6, -7)$,
 $\vec{e}'_3 = (3, 9, 5, 7)$, $\vec{e}'_4 = (2, 3, 2, 3)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -6x - 7y + 12z + 3w + 8 \\ -7x - 8y + 13z + 4w + 10 \\ 2x + 3y - 5z - w - 4 \\ -5x - 7y + 10z + 4w + 12 \end{pmatrix}$$

23. $O'(-1, 2, 2, -2)$, $\vec{e}'_1 = (-8, 10, 6, 15)$, $\vec{e}'_2 = (-7, 9, 6, 13)$,
 $\vec{e}'_3 = (-5, 6, 4, 9)$, $\vec{e}'_4 = (2, -5, -3, -6)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -x + 2y - 2z - w - 3 \\ -6x + 9y - 3z - 8w - 34 \\ 9x - 15y + 7z + 12w + 49 \\ -2x + 2y - z - 2w - 8 \end{pmatrix}$$

24. $O'(-3, -2, -3, 2)$, $\vec{e}'_1 = (-6, -2, 4, 3)$, $\vec{e}'_2 = (2, 1, -1, -1)$,
 $\vec{e}'_3 = (-6, -1, 4, 1)$, $\vec{e}'_4 = (-7, -1, 5, 2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 3x - 3y + 4z - w + 17 \\ 5x - 4y + 7z - 2w + 32 \\ 2x - 3y + 3z - 2w + 13 \\ -3x + 4y - 4z + 2w - 17 \end{pmatrix}$$

25. $O'(1, 1, 1, -3)$, $\vec{e}'_1 = (14, -15, 7, -8)$, $\vec{e}'_2 = (7, -5, 1, -3)$,
 $\vec{e}'_3 = (-6, 6, -2, 3)$, $\vec{e}'_4 = (-8, 10, -5, 5)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 5x + y + 6z + 12w + 24 \\ -5x - 6z - 14w - 31 \\ -5x + y - 5z - 15w - 36 \\ 8x + y + 9z + 20w + 42 \end{pmatrix}$$

26. $O'(-1, 1, 0, 2)$, $\vec{e}'_1 = (-13, 2, -8, 15)$, $\vec{e}'_2 = (-4, -1, -2, 5)$,
 $\vec{e}'_3 = (7, -4, 5, -7)$, $\vec{e}'_4 = (8, -2, 5, -9)$.

$$\text{ОТВЕТ: } \begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 4x - y - 5z + w + 3 \\ -6x - y + 2z - 4w + 3 \\ 3x + y + z + 3w - 4 \\ x - 3y - 8z - 3w + 10 \end{pmatrix}$$

27. $O'(3, -3, -3, 0)$, $\vec{e}'_1 = (12, 3, 5, -3)$, $\vec{e}'_2 = (9, 2, 9, -1)$,
 $\vec{e}'_3 = (-11, -3, -12, 1)$, $\vec{e}'_4 = (7, 2, 2, -2)$.

Ответ:
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 10x + y - 7z + 29w - 48 \\ -8x - 3y + 6z - 25w + 33 \\ -4x - 2y + 3z - 13w + 15 \\ -13x - y + 9z - 38w + 63 \end{pmatrix}$$

28. $O'(-3, 1, 1, -2)$, $\vec{e}'_1 = (-4, 9, -6, 16)$, $\vec{e}'_2 = (1, -3, 2, -5)$,
 $\vec{e}'_3 = (1, -3, 3, -6)$, $\vec{e}'_4 = (2, -4, 2, -7)$.

Ответ:
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -3x + y - 2z - 2w - 12 \\ -3x - 2y - z - 6 \\ -2x + 2y - z - 2w - 11 \\ -3x + 2y - 3z - 3w - 14 \end{pmatrix}$$

29. $O'(-2, 3, -3, 2)$, $\vec{e}'_1 = (-5, 17, -4, 9)$, $\vec{e}'_2 = (2, -7, 1, -4)$,
 $\vec{e}'_3 = (-6, 22, -3, 13)$, $\vec{e}'_4 = (-4, 14, -4, 7)$.

Ответ:
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} -5x - 4y - 2z + 4w - 12 \\ -x - 7y - 6z + 10w - 19 \\ x - y - z + 2w - 2 \\ 4x + 3y + z - 3w + 8 \end{pmatrix}$$

30. $O'(3, 0, 2, -2)$, $\vec{e}'_1 = (2, -6, -4, 3)$, $\vec{e}'_2 = (1, -8, -6, 4)$,
 $\vec{e}'_3 = (-3, 17, 12, -8)$, $\vec{e}'_4 = (-1, 5, 3, -2)$.

Ответ:
$$\begin{pmatrix} x' \\ y' \\ z' \\ w' \end{pmatrix} = \begin{pmatrix} 2z + 3w + 2 \\ -3x - y + 6z + 8w + 13 \\ -2x - y + 5z + 6w + 8 \\ 2x + 2y - 5z - 4w - 4 \end{pmatrix}$$

2. Евклидовы пространства

2.1. Задачи

Задача 2.1. Найти матрицу Грама системы векторов

a) $\vec{a}, \vec{b} \in \mathbb{R}^2$,

b) $\vec{a}, \vec{b}, \vec{c} \in \mathbb{R}^5$,

заданных своими координатами в ортонормированном базисе.

1. a) $\vec{a} = (-2, -1), \vec{b} = (1, 0)$;

b) $\vec{a} = (2, 2, 2, 1, 1), \vec{b} = (2, -2, -1, -1, 0), \vec{c} = (-2, 2, 1, 0, -2)$.

Ответ: $\begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} 14 & -3 & 0 \\ -3 & 10 & -9 \\ 0 & -9 & 13 \end{pmatrix}$

2. a) $\vec{a} = (3, 1), \vec{b} = (-2, 1)$;

b) $\vec{a} = (1, 2, 2, 1, 2), \vec{b} = (2, 0, 0, 0, 0), \vec{c} = (0, -2, -1, 1, 2)$.

Ответ: $\begin{pmatrix} 10 & -5 \\ -5 & 5 \end{pmatrix}, \begin{pmatrix} 14 & 2 & -1 \\ 2 & 4 & 0 \\ -1 & 0 & 10 \end{pmatrix}$

3. a) $\vec{a} = (3, -2), \vec{b} = (2, -1)$;

b) $\vec{a} = (2, 2, 1, 2, 1), \vec{b} = (-1, 0, -2, -2, 1), \vec{c} = (1, -2, -1, 2, -2)$.

Ответ: $\begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}, \begin{pmatrix} 14 & -7 & -1 \\ -7 & 10 & -5 \\ -1 & -5 & 14 \end{pmatrix}$

4. a) $\vec{a} = (2, -2), \vec{b} = (0, -3)$;

b) $\vec{a} = (2, 1, 2, 2, 2), \vec{b} = (2, -1, 2, 2, 1), \vec{c} = (2, -1, -1, 1, 0)$.

Ответ: $\begin{pmatrix} 8 & 6 \\ 6 & 9 \end{pmatrix}, \begin{pmatrix} 17 & 13 & 3 \\ 13 & 14 & 5 \\ 3 & 5 & 7 \end{pmatrix}$

5. a) $\vec{a} = (2, -2), \vec{b} = (1, 0)$;

b) $\vec{a} = (1, 2, 1, 2, 2), \vec{b} = (-1, 1, -1, 0, -2), \vec{c} = (1, 1, 1, 1, -2)$.

Ответ: $\begin{pmatrix} 8 & 2 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 14 & -4 & 2 \\ -4 & 7 & 3 \\ 2 & 3 & 8 \end{pmatrix}$

6. a) $\vec{a} = (3, -2)$, $\vec{b} = (2, -2)$;

b) $\vec{a} = (1, 1, 1, 1, 1)$, $\vec{b} = (1, 1, 0, 2, 1)$, $\vec{c} = (1, -2, -1, 1, 1)$.

Ответ: $\begin{pmatrix} 13 & 10 \\ 10 & 8 \end{pmatrix}$, $\begin{pmatrix} 5 & 5 & 0 \\ 5 & 7 & 2 \\ 0 & 2 & 8 \end{pmatrix}$

7. a) $\vec{a} = (1, -3)$, $\vec{b} = (2, 2)$;

b) $\vec{a} = (1, 2, 2, 1, 1)$, $\vec{b} = (-1, 2, 1, 1, 0)$, $\vec{c} = (-2, -2, -1, 1, -2)$.

Ответ: $\begin{pmatrix} 10 & -4 \\ -4 & 8 \end{pmatrix}$, $\begin{pmatrix} 11 & 6 & -9 \\ 6 & 7 & -2 \\ -9 & -2 & 14 \end{pmatrix}$

8. a) $\vec{a} = (2, -1)$, $\vec{b} = (1, 1)$;

b) $\vec{a} = (2, 2, 2, 2, 1)$, $\vec{b} = (2, -2, 2, 1, 2)$, $\vec{c} = (-1, -2, 1, 1, 0)$.

Ответ: $\begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}$, $\begin{pmatrix} 17 & 8 & -2 \\ 8 & 17 & 5 \\ -2 & 5 & 7 \end{pmatrix}$

9. a) $\vec{a} = (-2, 3)$, $\vec{b} = (-1, 2)$;

b) $\vec{a} = (2, 2, 1, 2, 2)$, $\vec{b} = (-2, -2, 2, -2, 0)$, $\vec{c} = (2, -2, -2, 0, -1)$.

Ответ: $\begin{pmatrix} 13 & 8 \\ 8 & 5 \end{pmatrix}$, $\begin{pmatrix} 17 & -10 & -4 \\ -10 & 16 & -4 \\ -4 & -4 & 13 \end{pmatrix}$

10. a) $\vec{a} = (-3, -3)$, $\vec{b} = (3, 1)$;

b) $\vec{a} = (2, 1, 1, 1, 2)$, $\vec{b} = (2, -1, 0, 0, -1)$, $\vec{c} = (-2, 1, 2, 2, 1)$.

Ответ: $\begin{pmatrix} 18 & -12 \\ -12 & 10 \end{pmatrix}$, $\begin{pmatrix} 11 & 1 & 3 \\ 1 & 6 & -6 \\ 3 & -6 & 14 \end{pmatrix}$

11. a) $\vec{a} = (-2, 3)$, $\vec{b} = (0, 3)$;

b) $\vec{a} = (2, 1, 1, 2, 1)$, $\vec{b} = (2, -2, 1, 0, 2)$, $\vec{c} = (-1, -2, -1, 2, 0)$.

Ответ: $\begin{pmatrix} 13 & 9 \\ 9 & 9 \end{pmatrix}$, $\begin{pmatrix} 11 & 5 & -1 \\ 5 & 13 & 1 \\ -1 & 1 & 10 \end{pmatrix}$

12. a) $\vec{a} = (1, 2)$, $\vec{b} = (-2, -1)$;

b) $\vec{a} = (1, 1, 2, 1, 1)$, $\vec{b} = (-1, -1, -1, -1, -1)$, $\vec{c} = (2, -1, -2, 0, -2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}, \begin{pmatrix} 8 & -6 & -5 \\ -6 & 5 & 3 \\ -5 & 3 & 13 \end{pmatrix}$$

13. a) $\vec{a} = (2, -1)$, $\vec{b} = (3, -2)$;
 b) $\vec{a} = (1, 1, 1, 2, 2)$, $\vec{b} = (2, 0, -1, -1, 0)$, $\vec{c} = (2, 2, 0, -2, 1)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}, \begin{pmatrix} 11 & -1 & 2 \\ -1 & 6 & 6 \\ 2 & 6 & 13 \end{pmatrix}$$

14. a) $\vec{a} = (0, 2)$, $\vec{b} = (1, -3)$;
 b) $\vec{a} = (1, 1, 2, 1, 1)$, $\vec{b} = (1, -1, 2, 2, 2)$, $\vec{c} = (1, 1, 1, 2, 2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 4 & -6 \\ -6 & 10 \end{pmatrix}, \begin{pmatrix} 8 & 8 & 8 \\ 8 & 14 & 10 \\ 8 & 10 & 11 \end{pmatrix}$$

15. a) $\vec{a} = (0, -1)$, $\vec{b} = (-3, 2)$;
 b) $\vec{a} = (2, 2, 2, 1, 2)$, $\vec{b} = (1, 2, -1, 1, -1)$, $\vec{c} = (0, 1, -2, 0, 0)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 1 & -2 \\ -2 & 13 \end{pmatrix}, \begin{pmatrix} 17 & 3 & -2 \\ 3 & 8 & 4 \\ -2 & 4 & 5 \end{pmatrix}$$

16. a) $\vec{a} = (0, -2)$, $\vec{b} = (3, -2)$;
 b) $\vec{a} = (1, 2, 2, 2, 2)$, $\vec{b} = (-1, 2, 0, 1, 1)$, $\vec{c} = (-2, -2, 0, 1, -2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 4 & 4 \\ 4 & 13 \end{pmatrix}, \begin{pmatrix} 17 & 7 & -8 \\ 7 & 7 & -3 \\ -8 & -3 & 13 \end{pmatrix}$$

17. a) $\vec{a} = (-2, 3)$, $\vec{b} = (-2, 2)$;
 b) $\vec{a} = (2, 2, 2, 1, 1)$, $\vec{b} = (0, 0, -2, 2, 1)$, $\vec{c} = (-2, 0, 1, 0, 2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 13 & 10 \\ 10 & 8 \end{pmatrix}, \begin{pmatrix} 14 & -1 & 0 \\ -1 & 9 & 0 \\ 0 & 0 & 9 \end{pmatrix}$$

18. a) $\vec{a} = (1, -3)$, $\vec{b} = (3, 0)$;
 b) $\vec{a} = (1, 2, 2, 1, 1)$, $\vec{b} = (-2, 1, 1, -1, -2)$, $\vec{c} = (-2, -2, 0, 0, 0)$.

$$\text{Ответ: } \begin{pmatrix} 10 & 3 \\ 3 & 9 \end{pmatrix}, \begin{pmatrix} 11 & -1 & -6 \\ -1 & 11 & 2 \\ -6 & 2 & 8 \end{pmatrix}$$

19. a) $\vec{a} = (3, 3)$, $\vec{b} = (-2, 0)$;
 b) $\vec{a} = (1, 2, 2, 1, 2)$, $\vec{b} = (0, -2, -2, 1, 2)$, $\vec{c} = (0, 2, 0, -1, -2)$.

$$\text{Ответ: } \begin{pmatrix} 18 & -6 \\ -6 & 4 \end{pmatrix}, \begin{pmatrix} 14 & -3 & -1 \\ -3 & 13 & -9 \\ -1 & -9 & 9 \end{pmatrix}$$

20. a) $\vec{a} = (-1, 3)$, $\vec{b} = (3, 0)$;
 b) $\vec{a} = (1, 1, 1, 2, 1)$, $\vec{b} = (0, 0, 1, -2, -1)$, $\vec{c} = (2, -2, -1, -1, 2)$.

$$\text{Ответ: } \begin{pmatrix} 10 & -3 \\ -3 & 9 \end{pmatrix}, \begin{pmatrix} 8 & -4 & -1 \\ -4 & 6 & -1 \\ -1 & -1 & 14 \end{pmatrix}$$

21. a) $\vec{a} = (-2, -2)$, $\vec{b} = (-1, 2)$;
 b) $\vec{a} = (1, 2, 1, 1, 1)$, $\vec{b} = (0, 0, 2, -2, 1)$, $\vec{c} = (1, 2, -1, -2, 0)$.

$$\text{Ответ: } \begin{pmatrix} 8 & -2 \\ -2 & 5 \end{pmatrix}, \begin{pmatrix} 8 & 1 & 2 \\ 1 & 9 & 2 \\ 2 & 2 & 10 \end{pmatrix}$$

22. a) $\vec{a} = (0, 3)$, $\vec{b} = (-2, -2)$;
 b) $\vec{a} = (2, 1, 2, 1, 2)$, $\vec{b} = (0, 2, -1, 2, 0)$, $\vec{c} = (-1, 2, 1, 2, 0)$.

$$\text{Ответ: } \begin{pmatrix} 9 & -6 \\ -6 & 8 \end{pmatrix}, \begin{pmatrix} 14 & 2 & 4 \\ 2 & 9 & 7 \\ 4 & 7 & 10 \end{pmatrix}$$

23. a) $\vec{a} = (-1, 3)$, $\vec{b} = (-2, 3)$;
 b) $\vec{a} = (1, 1, 2, 1, 1)$, $\vec{b} = (-2, -2, 0, -2, -1)$, $\vec{c} = (2, -2, -1, 0, 1)$.

$$\text{Ответ: } \begin{pmatrix} 10 & 11 \\ 11 & 13 \end{pmatrix}, \begin{pmatrix} 8 & -7 & -1 \\ -7 & 13 & -1 \\ -1 & -1 & 10 \end{pmatrix}$$

24. a) $\vec{a} = (-1, 3)$, $\vec{b} = (1, 1)$;
 b) $\vec{a} = (1, 2, 1, 2, 1)$, $\vec{b} = (-2, 0, -1, 1, 1)$, $\vec{c} = (2, -2, -1, 2, -2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 10 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 11 & 0 & -1 \\ 0 & 7 & -3 \\ -1 & -3 & 17 \end{pmatrix}$$

25. a) $\vec{a} = (3, -2)$, $\vec{b} = (3, 0)$;
 b) $\vec{a} = (1, 2, 1, 2, 2)$, $\vec{b} = (1, 1, -1, 0, 1)$, $\vec{c} = (1, -2, 0, -1, 0)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 13 & 9 \\ 9 & 9 \end{pmatrix}, \begin{pmatrix} 14 & 4 & -5 \\ 4 & 4 & -1 \\ -5 & -1 & 6 \end{pmatrix}$$

26. a) $\vec{a} = (-1, 0)$, $\vec{b} = (2, 1)$;
 b) $\vec{a} = (1, 1, 2, 1, 2)$, $\vec{b} = (-2, -2, 0, 0, -2)$, $\vec{c} = (2, -2, -2, 1, 1)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}, \begin{pmatrix} 11 & -8 & -1 \\ -8 & 12 & -2 \\ -1 & -2 & 14 \end{pmatrix}$$

27. a) $\vec{a} = (2, 2)$, $\vec{b} = (0, 2)$;
 b) $\vec{a} = (1, 1, 1, 1, 1)$, $\vec{b} = (-2, 1, -2, -2, -2)$, $\vec{c} = (-1, -2, -2, 0, -2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 8 & 4 \\ 4 & 4 \end{pmatrix}, \begin{pmatrix} 5 & -7 & -7 \\ -7 & 17 & 8 \\ -7 & 8 & 13 \end{pmatrix}$$

28. a) $\vec{a} = (0, 3)$, $\vec{b} = (-1, 1)$;
 b) $\vec{a} = (2, 1, 2, 1, 1)$, $\vec{b} = (2, -1, -1, 0, -1)$, $\vec{c} = (1, -2, -1, 1, -2)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 9 & 3 \\ 3 & 2 \end{pmatrix}, \begin{pmatrix} 11 & 0 & -3 \\ 0 & 7 & 7 \\ -3 & 7 & 11 \end{pmatrix}$$

29. a) $\vec{a} = (1, 3)$, $\vec{b} = (-1, 2)$;
 b) $\vec{a} = (2, 2, 1, 1, 2)$, $\vec{b} = (-2, -2, -2, -2, -1)$, $\vec{c} = (2, 2, -2, -1, -1)$.

$$\text{ОТВЕТ: } \begin{pmatrix} 10 & 5 \\ 5 & 5 \end{pmatrix}, \begin{pmatrix} 14 & -14 & 3 \\ -14 & 17 & -1 \\ 3 & -1 & 14 \end{pmatrix}$$

30. a) $\vec{a} = (-2, 0)$, $\vec{b} = (-1, -1)$;
 b) $\vec{a} = (1, 1, 1, 2, 1)$, $\vec{b} = (1, 2, -2, 0, 2)$, $\vec{c} = (1, 0, -2, -1, -1)$.

$$\text{Ответ: } \begin{pmatrix} 4 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 8 & 3 & -4 \\ 3 & 13 & 3 \\ -4 & 3 & 7 \end{pmatrix}$$

Задача 2.2. Применяя процесс ортогонализации, найти ортонормированный базис подпространства $\langle \vec{u}, \vec{v}, \vec{w} \rangle$.

1. $\vec{u} = (2, 2, -2, 0)$, $\vec{v} = (-2, 2, 0, -1)$, $\vec{w} = (1, -2, 0, 1)$.

Ответ: $3/2(-2/9, -1/9, -1/3, 2/9)\sqrt{2}$, $1/6(2, 2, -2, 0)\sqrt{3}$, $(-2/3, 2/3, 0, -1/3)$

2. $\vec{u} = (-2, 2, 0, 0)$, $\vec{v} = (2, 1, 0, -2)$, $\vec{w} = (0, -1, -1, -1)$.

Ответ: $1/42(-\frac{10}{17}, -\frac{10}{17}, -1, -\frac{15}{17})\sqrt{714}$, $\frac{3}{34}(-\frac{14}{9}, \frac{20}{9}, 0, -4/9)\sqrt{17}$, $(2/3, 1/3, 0, -2/3)$

3. $\vec{u} = (-3, 0, -1, 1)$, $\vec{v} = (-1, -3, 1, -3)$, $\vec{w} = (0, 0, -3, 1)$.

Ответ: $\frac{2}{219}(-\frac{61}{20}, -\frac{3}{20}, -\frac{19}{20}, \frac{17}{20})\sqrt{1095}$, $\frac{1}{1522}(\frac{160}{219}, -\frac{62}{73}, -\frac{521}{219}, -\frac{41}{219})\sqrt{333318}$, $1/1(-1, -3, 1, -3)\sqrt{5}$

4. $\vec{u} = (0, 1, -3, 3)$, $\vec{v} = (-1, 1, 3, 1)$, $\vec{w} = (3, -3, -1, 1)$.

Ответ: $\frac{2}{203}(-\frac{5}{12}, \frac{17}{12}, -7/4, \frac{41}{12})\sqrt{609}$, $\frac{1}{744}(\frac{472}{203}, -\frac{468}{203}, \frac{28}{29}, \frac{352}{203})\sqrt{37758}$, $1/6(-1, 1, 3, 1)\sqrt{3}$

5. $\vec{u} = (0, -3, 3, 0)$, $\vec{v} = (-1, 1, 0, -1)$, $\vec{w} = (0, -2, 3, 3)$.

Ответ: $1/15(-1, -2, 3, -1)\sqrt{15}$, $1/3(-1, 1, 0, -1)\sqrt{3}$, $1/7(-1, 1, 1, 2)\sqrt{7}$

6. $\vec{u} = (-2, -1, 0, -1)$, $\vec{v} = (-2, -1, 1, -3)$, $\vec{w} = (3, -3, -1, -3)$.

Ответ: $1/26(-\frac{14}{15}, -\frac{7}{15}, -\frac{8}{15}, 3/5)\sqrt{390}$, $1/17(\frac{29}{13}, -\frac{44}{13}, -\frac{28}{13}, -\frac{14}{13})\sqrt{13}$, $1/15(-2, -1, 1, -3)\sqrt{15}$

7. $\vec{u} = (-3, 0, -1, -3)$, $\vec{v} = (-2, -1, 1, 0)$, $\vec{w} = (-2, 3, 3, 1)$.

Ответ: $\frac{1}{89}(-4/3, 5/6, -\frac{11}{6}, -3)\sqrt{534}$, $1/6(-2, -1, 1, 0)\sqrt{6}$, $\frac{1}{1743}(-\frac{86}{89}, \frac{343}{89}, \frac{171}{89}, \frac{2}{89})\sqrt{13}$

8. $\vec{u} = (-2, -2, -1, 1)$, $\vec{v} = (-1, -2, -2, -2)$, $\vec{w} = (2, 0, -2, -2)$.

Ответ: $\frac{1}{94}(-\frac{20}{13}, -\frac{14}{13}, -1/13, \frac{25}{13})\sqrt{1222}$, $\frac{1}{68}(\frac{48}{47}, -\frac{4}{47}, -\frac{54}{47}, \frac{34}{47})\sqrt{1598}$, $1/13(-1, -2, -2, -2)\sqrt{13}$

9. $\vec{u} = (3, 0, -1, -3)$, $\vec{v} = (1, 3, 1, -2)$, $\vec{w} = (-3, -1, 1, 0)$.

Ответ: $\frac{1}{221} (\frac{37}{15}, -8/5, -\frac{23}{15}, -\frac{29}{15})\sqrt{3315}$, $\frac{1}{628} (-\frac{318}{221}, -\frac{176}{221}, \frac{126}{221}, -\frac{360}{221})\sqrt{69394}$, $1/1(-2)\sqrt{15}$

10. $\vec{u} = (-3, 2, -3, -3)$, $\vec{v} = (-3, -2, -3, -1)$, $\vec{w} = (3, 0, -3, 3)$.

Ответ: $\frac{1}{426} (\frac{123}{53}, \frac{54}{53}, -\frac{195}{53}, \frac{108}{53})\sqrt{7526}$, $1/23(-3, -2, -3, -1)\sqrt{23}$, $\frac{1}{212} (-\frac{18}{23}, \frac{80}{23}, -\frac{18}{23}, -\frac{52}{23})\sqrt{2438}$

11. $\vec{u} = (-3, 0, -3, 0)$, $\vec{v} = (3, 2, 0, -2)$, $\vec{w} = (-1, -3, -3, -2)$.

Ответ: $1/15(-\frac{24}{17}, \frac{18}{17}, -3, -\frac{18}{17})\sqrt{17}$, $1/17(3, 2, 0, -2)\sqrt{17}$, $\frac{5}{373} (\frac{22}{25}, -\frac{79}{25}, -\frac{22}{25}, -\frac{46}{25})\sqrt{373}$

12. $\vec{u} = (2, 3, -1, -1)$, $\vec{v} = (2, -2, 2, 3)$, $\vec{w} = (1, 2, 3, 2)$.

Ответ: $1/38(8/3, 7/3, -1/3, 0)\sqrt{114}$, $\frac{1}{141} (-\frac{9}{7}, \frac{25}{14}, \frac{31}{14}, 4/7)\sqrt{1974}$, $1/21(2, -2, 2, 3)\sqrt{21}$

13. $\vec{u} = (2, 0, 1, 3)$, $\vec{v} = (0, 3, 2, 0)$, $\vec{w} = (1, -1, -3, 3)$.

Ответ: $\frac{1}{653} (-\frac{33}{89}, \frac{124}{89}, -\frac{186}{89}, \frac{84}{89})\sqrt{58117}$, $\frac{1}{178} (2, -\frac{6}{13}, \frac{9}{13}, 3)\sqrt{2314}$, $1/13(0, 3, 2, 0)$

14. $\vec{u} = (-3, 2, 0, -3)$, $\vec{v} = (-2, -1, 0, 0)$, $\vec{w} = (2, -2, 1, 1)$.

Ответ: $1/5(-2, -1, 0, 0)\sqrt{5}$, $\frac{1}{215} (\frac{33}{94}, -\frac{33}{47}, 1, -\frac{77}{94})\sqrt{20210}$, $\frac{1}{94} (-7/5, \frac{14}{5}, 0, -3)\sqrt{470}$

15. $\vec{u} = (2, 1, 1, 0)$, $\vec{v} = (-2, -1, -3, 1)$, $\vec{w} = (2, 1, -3, 1)$.

Ответ: $1/26(\frac{14}{15}, \frac{7}{15}, -3/5, \frac{8}{15})\sqrt{390}$, $1/15(-2, -1, -3, 1)\sqrt{15}$, $1/10(2/13, 1/13, -\frac{5}{13}, -\frac{10}{13})\sqrt{130}$

16. $\vec{u} = (0, -3, -3, 1)$, $\vec{v} = (3, 2, -1, -3)$, $\vec{w} = (-1, 0, -2, -1)$.

Ответ: $\frac{1}{1635} (-\frac{605}{401}, \frac{245}{401}, -\frac{353}{401}, -\frac{324}{401})\sqrt{655635}$, $1/23(3, 2, -1, -3)\sqrt{23}$, $\frac{1}{401} (\frac{18}{23}, -\frac{57}{23}, -\frac{75}{23}, \frac{5}{23})\sqrt{9223}$

17. $\vec{u} = (2, 3, 1, -1)$, $\vec{v} = (-3, 3, -1, 1)$, $\vec{w} = (-2, 3, -2, 1)$.

Ответ: $\frac{1}{514} (\frac{162}{299}, \frac{27}{299}, -\frac{352}{299}, \frac{53}{299})\sqrt{153686}$, $\frac{2}{299} (\frac{43}{20}, \frac{57}{20}, \frac{21}{20}, -\frac{21}{20})\sqrt{1495}$, $1/10(-3, 3, -1, 1)\sqrt{5}$

18. $\vec{u} = (-1, 2, 2, -3)$, $\vec{v} = (-3, 3, -2, -1)$, $\vec{w} = (-1, 2, 0, 3)$.

Ответ: $\frac{1}{70} (1/23, \frac{22}{23}, \frac{62}{23}, -\frac{61}{23})\sqrt{322}$, $1/23 (-3, 3, -2, -1)\sqrt{23}$, $1/10 (-1/5, 8/5, 8/5, \frac{11}{5})\sqrt{10}$

19. $\vec{u} = (-2, -3, 0, 0)$, $\vec{v} = (0, 0, -2, -1)$, $\vec{w} = (-3, 1, -1, -2)$.

Ответ: $1/5 (0, 0, -2, -1)\sqrt{5}$, $1/13 (-2, -3, 0, 0)\sqrt{13}$, $1/38 (-\frac{33}{13}, \frac{22}{13}, 3/5, -6/5)\sqrt{130}$

20. $\vec{u} = (-1, -3, -2, 3)$, $\vec{v} = (1, 1, -1, -1)$, $\vec{w} = (0, -2, -2, 3)$.

Ответ: $\frac{2}{67} (1/4, -7/4, -\frac{13}{4}, 7/4)\sqrt{67}$, $(1/2, 1/2, -1/2, -1/2)$, $\frac{1}{58} (\frac{35}{67}, \frac{23}{67}, \frac{14}{67}, \frac{44}{67})\sqrt{67}$

21. $\vec{u} = (-3, -1, -3, -1)$, $\vec{v} = (-1, -3, 2, 1)$, $\vec{w} = (2, 1, -2, 0)$.

Ответ: $1/15 (-1, -3, 2, 1)\sqrt{15}$, $\frac{1}{1038} (\frac{15}{13}, -\frac{268}{299}, -\frac{308}{299}, \frac{157}{299})\sqrt{310362}$, $\frac{1}{299} (-\frac{46}{15}, -6/5, -\frac{43}{15}, -\frac{14}{15})\sqrt{4485}$

22. $\vec{u} = (-1, 2, -2, -2)$, $\vec{v} = (-3, -2, -1, 1)$, $\vec{w} = (-1, 3, -1, 2)$.

Ответ: $\frac{1}{194} (-6/5, \frac{28}{15}, -\frac{31}{15}, -\frac{29}{15})\sqrt{2910}$, $\frac{1}{195} (-\frac{52}{97}, \frac{221}{97}, -\frac{39}{194}, \frac{533}{194})\sqrt{2910}$, $1/15 (-3, -2, -1, 1)\sqrt{15}$

23. $\vec{u} = (3, -3, -3, 2)$, $\vec{v} = (-2, 3, 2, 0)$, $\vec{w} = (-1, 0, -2, -1)$.

Ответ: $\frac{1}{86} (\frac{9}{17}, \frac{12}{17}, -\frac{9}{17}, 2)\sqrt{1462}$, $\frac{1}{153} (-\frac{93}{86}, \frac{24}{43}, -\frac{165}{86}, -\frac{18}{43})\sqrt{4386}$, $1/17 (-2, 3, 2, 0)\sqrt{17}$

24. $\vec{u} = (-3, 2, -1, -2)$, $\vec{v} = (-1, -2, -1, -2)$, $\vec{w} = (-1, -2, 3, -3)$.

Ответ: $\frac{1}{677} (-\frac{16}{41}, -\frac{8}{41}, \frac{154}{41}, -\frac{61}{41})\sqrt{27757}$, $\frac{1}{82} (-\frac{13}{5}, \frac{14}{5}, -3/5, -6/5)\sqrt{410}$, $1/10 (-1, -2, -1, -2)\sqrt{10}$

25. $\vec{u} = (-3, 1, 3, -1)$, $\vec{v} = (3, 0, -2, 2)$, $\vec{w} = (0, -3, -1, -2)$.

Ответ: $1/3 (0, 1, 1, 1)\sqrt{3}$, $1/30 (\frac{6}{17}, -1, \frac{13}{17}, \frac{4}{17})\sqrt{510}$, $1/17 (3, 0, -2, 2)\sqrt{17}$

26. $\vec{u} = (-2, 3, 0, 3)$, $\vec{v} = (0, 2, -2, 1)$, $\vec{w} = (-3, 3, -3, 3)$.

Ответ: $(0, 2/3, -2/3, 1/3)$, $1/13 (-2, 1, 2, 2)\sqrt{13}$, $\frac{1}{62} (-\frac{21}{13}, -\frac{40}{39}, -\frac{41}{39}, -\frac{2}{39})\sqrt{806}$

27. $\vec{u} = (-3, -1, 3, 2)$, $\vec{v} = (1, -3, -2, -3)$, $\vec{w} = (1, -3, 2, 1)$.

Ответ: $\frac{1}{385} \left(-\frac{57}{23}, -\frac{59}{23}, \frac{45}{23}, \frac{10}{23}\right) \sqrt{8855}$, $1/23 (1, -3, -2, -3) \sqrt{23}$, $1/8 \left(\frac{16}{7}, -\frac{8}{7}, \frac{8}{7}, \frac{8}{7}\right) \sqrt{7}$

28. $\vec{u} = (-3, -2, -2, 3)$, $\vec{v} = (1, 1, 2, 3)$, $\vec{w} = (3, -1, 0, -3)$.

Ответ: $1/15 (1, 1, 2, 3) \sqrt{15}$, $\frac{1}{574} \left(\frac{316}{195}, -\frac{344}{195}, -\frac{58}{195}, \frac{16}{65}\right) \sqrt{55965}$, $1/26 (-3, -2, -2, 3) \sqrt{26}$

29. $\vec{u} = (1, 3, -2, 1)$, $\vec{v} = (2, 1, 2, 2)$, $\vec{w} = (-3, -2, -3, -1)$.

Ответ: $1/13 (2, 1, 2, 2) \sqrt{13}$, $\frac{1}{186} \left(\frac{7}{13}, \frac{36}{13}, -\frac{32}{13}, \frac{7}{13}\right) \sqrt{2418}$, $\frac{1}{307} \left(-\frac{49}{93}, -\frac{22}{31}, -\frac{55}{93}, \frac{137}{93}\right) \sqrt{28551}$

30. $\vec{u} = (-1, -1, -1, 0)$, $\vec{v} = (3, 2, -1, 1)$, $\vec{w} = (3, -2, -1, 3)$.

Ответ: $\frac{1}{212} \left(\frac{42}{29}, -\frac{76}{29}, \frac{34}{29}, \frac{60}{29}\right) \sqrt{3074}$, $1/15 (3, 2, -1, 1) \sqrt{15}$, $1/29 (-1/5, -\frac{7}{15}, -\frac{19}{15}, \frac{4}{15}) \sqrt{435}$

Задача 2.3. Найти длину стороны AB и $\cos \angle ACB$ треугольника $\triangle ABC$, если известна матрица метрических коэффициентов базиса G .

1. $G = \begin{pmatrix} 3 & -2 \\ -2 & 4 \end{pmatrix}$; $A(-1, -14)$, $B(5, 5)$, $C(8, -4)$.

Ответ: $AB = 2\sqrt{274}$, $\cos = -\frac{59}{14433} \sqrt{14433}$

2. $G = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$; $A(7, 3)$, $B(10, 1)$, $C(6, 9)$.

Ответ: $AB = \sqrt{34}$, $\cos = \frac{11}{25} \sqrt{5}$

3. $G = \begin{pmatrix} 4 & -1 \\ -1 & 5 \end{pmatrix}$; $A(-11, -10)$, $B(-13, -3)$, $C(-8, -6)$.

Ответ: $AB = 17$, $\cos = -\frac{11}{1610} \sqrt{161}$

4. $G = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$; $A(-7, -1)$, $B(-17, -6)$, $C(-9, 1)$.

Ответ: $AB = 5\sqrt{10}$, $\cos = \frac{7}{274} \sqrt{274}$

5. $G = \begin{pmatrix} 5 & 2 \\ 2 & 4 \end{pmatrix}$; $A(7, -15)$, $B(9, -6)$, $C(-1, -8)$.

Ответ: $AB = 4\sqrt{26}$, $\cos = \frac{59}{10877}\sqrt{10877}$

6. $G = \begin{pmatrix} 3 & -2 \\ -2 & 4 \end{pmatrix}$; $A(-9, -1)$, $B(-2, -9)$, $C(-3, -5)$.

Ответ: $AB = \sqrt{627}$, $\cos = -\frac{69}{5561}\sqrt{5561}$

7. $G = \begin{pmatrix} 4 & 3 \\ 3 & 5 \end{pmatrix}$; $A(5, -13)$, $B(3, -3)$, $C(-4, -5)$.

Ответ: $AB = 6\sqrt{11}$, $\cos = \frac{29}{1590}\sqrt{159}$

8. $G = \begin{pmatrix} 5 & -2 \\ -2 & 4 \end{pmatrix}$; $A(5, -9)$, $B(-4, -9)$, $C(3, -8)$.

Ответ: $AB = 9\sqrt{5}$, $\cos = -\frac{19}{442}\sqrt{442}$

9. $G = \begin{pmatrix} 5 & -2 \\ -2 & 1 \end{pmatrix}$; $A(2, 6)$, $B(-5, 10)$, $C(-2, 8)$.

Ответ: $AB = \sqrt{373}$, $\cos = -\frac{46}{2117}\sqrt{2117}$

10. $G = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$; $A(-9, 6)$, $B(0, 14)$, $C(-8, 7)$.

Ответ: $AB = \sqrt{113}$, $\cos = -\frac{13}{170}\sqrt{170}$

11. $G = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$; $A(-18, 7)$, $B(1, 3)$, $C(-8, 6)$.

Ответ: $AB = \sqrt{745}$, $\cos = -\frac{61}{3770}\sqrt{3770}$

12. $G = \begin{pmatrix} 5 & -1 \\ -1 & 5 \end{pmatrix}$; $A(-1, -10)$, $B(6, -10)$, $C(3, -2)$.

Ответ: $AB = 7\sqrt{5}$, $\cos = \frac{3}{59}\sqrt{177}$

13. $G = \begin{pmatrix} 4 & -1 \\ -1 & 3 \end{pmatrix}$; $A(7, -9)$, $B(10, -10)$, $C(2, -5)$.

Ответ: $AB = 3\sqrt{5}$, $\cos = \frac{277}{38634}\sqrt{19317}$

$$14. G = \begin{pmatrix} 4 & 2 \\ 2 & 5 \end{pmatrix}; \quad A(1, 6), B(11, -8), C(3, -4).$$

$$\text{ОТВЕТ: } AB = 2\sqrt{205}, \cos = -\frac{11}{1417}\sqrt{1417}$$

$$15. G = \begin{pmatrix} 4 & -3 \\ -3 & 5 \end{pmatrix}; \quad A(5, -15), B(4, -4), C(8, -5).$$

$$\text{ОТВЕТ: } AB = 15\sqrt{3}, \cos = -\frac{113}{16554}\sqrt{8277}$$

$$16. G = \begin{pmatrix} 3 & 2 \\ 2 & 3 \end{pmatrix}; \quad A(-10, 3), B(-12, -4), C(-4, 6).$$

$$\text{ОТВЕТ: } AB = \sqrt{215}, \cos = \frac{67}{4669}\sqrt{4669}$$

$$17. G = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}; \quad A(6, -10), B(-6, -9), C(-4, -4).$$

$$\text{ОТВЕТ: } AB = 2\sqrt{67}, \cos = \frac{3}{32}\sqrt{7}$$

$$18. G = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}; \quad A(9, 17), B(10, 16), C(6, 9).$$

$$\text{ОТВЕТ: } AB = \sqrt{11}, \cos = \frac{3}{55}\sqrt{330}$$

$$19. G = \begin{pmatrix} 3 & -3 \\ -3 & 5 \end{pmatrix}; \quad A(1, 15), B(0, 8), C(-5, 10).$$

$$\text{ОТВЕТ: } AB = \sqrt{206}, \cos = \frac{1}{8215}\sqrt{8215}$$

$$20. G = \begin{pmatrix} 5 & 2 \\ 2 & 2 \end{pmatrix}; \quad A(-4, -10), B(-1, -14), C(-8, -6).$$

$$\text{ОТВЕТ: } AB = \sqrt{29}, \cos = \frac{7}{149}\sqrt{447}$$

$$21. G = \begin{pmatrix} 4 & 2 \\ 2 & 4 \end{pmatrix}; \quad A(-5, 1), B(-6, -4), C(-9, 6).$$

$$\text{ОТВЕТ: } AB = 2\sqrt{31}, \cos = \frac{23}{1106}\sqrt{1659}$$

$$22. G = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}; \quad A(5, 18), B(-4, 2), C(3, 9).$$

$$\text{ОТВЕТ: } AB = 2\sqrt{281}, \cos = -\frac{37}{428}\sqrt{107}$$

$$23. G = \begin{pmatrix} 5 & -3 \\ -3 & 4 \end{pmatrix}; \quad A(-3, 1), B(-2, 9), C(5, 10).$$

$$\text{Ответ: } AB = \sqrt{213}, \cos = \frac{103}{7314} \sqrt{1219}$$

$$24. G = \begin{pmatrix} 2 & -2 \\ -2 & 4 \end{pmatrix}; \quad A(3, 2), B(-14, 3), C(-6, -5).$$

$$\text{Ответ: } AB = 5\sqrt{26}, \cos = \frac{3}{265} \sqrt{265}$$

$$25. G = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}; \quad A(-2, 9), B(10, 11), C(6, 6).$$

$$\text{Ответ: } AB = 6\sqrt{11}, \cos = -\frac{23}{22161} \sqrt{22161}$$

$$26. G = \begin{pmatrix} 3 & 3 \\ 3 & 4 \end{pmatrix}; \quad A(1, 7), B(-8, -3), C(-6, 3).$$

$$\text{Ответ: } AB = 13\sqrt{7}, \cos = -\frac{48}{7201} \sqrt{21603}$$

$$27. G = \begin{pmatrix} 4 & 2 \\ 2 & 3 \end{pmatrix}; \quad A(-1, -16), B(-2, -8), C(-8, -6).$$

$$\text{Ответ: } AB = 2\sqrt{41}, \cos = \frac{10}{27} \sqrt{2}$$

$$28. G = \begin{pmatrix} 5 & -3 \\ -3 & 2 \end{pmatrix}; \quad A(9, 8), B(7, 3), C(3, 6).$$

$$\text{Ответ: } AB = \sqrt{10}, \cos = \frac{69}{4930} \sqrt{4930}$$

$$29. G = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}; \quad A(4, -11), B(2, 3), C(7, -1).$$

$$\text{Ответ: } AB = 8\sqrt{13}, \cos = -\frac{11}{12325} \sqrt{2465}$$

$$30. G = \begin{pmatrix} 4 & 3 \\ 3 & 4 \end{pmatrix}; \quad A(6, -3), B(10, -11), C(5, -4).$$

$$\text{Ответ: } AB = 8\sqrt{2}, \cos = -1/43 \sqrt{301}$$

Задача 2.4. Найти площадь треугольника $\triangle ABC$ с вершинами A, B, C и координаты точки H — основания высоты, опущенной из точки B на сторону AC .

1. $A(0, 1, -5, -2, 4)$, $B(4, 2, -1, 1, 5)$, $C(5, 4, -4, 2, 7)$.
Ответ: $S = 2\sqrt{51}$, $H(7/2, \frac{31}{10}, -\frac{43}{10}, 4/5, \frac{61}{10})$.
2. $A(-1, -2, -7, -9, 6)$, $B(1, 1, -5, -7, 7)$, $C(4, 3, -5, -4, 8)$.
Ответ: $S = 1/2\sqrt{145}$, $H(\frac{122}{83}, \frac{39}{83}, -\frac{499}{83}, -\frac{542}{83}, \frac{580}{83})$.
3. $A(-7, 1, -7, 0, 2)$, $B(-5, 5, -2, 3, 6)$, $C(-3, 4, -3, 2, 5)$.
Ответ: $S = 2\sqrt{26}$, $H(-\frac{73}{27}, \frac{38}{9}, -\frac{73}{27}, \frac{58}{27}, \frac{47}{9})$.
4. $A(-2, 8, -7, 9, 6)$, $B(2, 11, -2, 10, 11)$, $C(-1, 13, -3, 12, 11)$.
Ответ: $S = 3/2\sqrt{143}$, $H(-\frac{85}{76}, \frac{943}{76}, -\frac{66}{19}, \frac{885}{76}, \frac{791}{76})$.
5. $A(2, -2, -2, -4, 8)$, $B(7, 3, 2, -1, 12)$, $C(3, 3, 0, 0, 12)$.
Ответ: $S = 1/2\sqrt{1286}$, $H(\frac{95}{31}, \frac{103}{31}, \frac{4}{31}, \frac{8}{31}, \frac{380}{31})$.
6. $A(5, 7, 4, 8, 8)$, $B(6, 11, 9, 11, 11)$, $C(6, 8, 5, 10, 10)$.
Ответ: $S = 2\sqrt{11}$, $H(7, 9, 6, 12, 12)$.
7. $A(-7, 0, -4, 8, 10)$, $B(-6, 4, 1, 10, 13)$, $C(-5, 2, 0, 11, 12)$.
Ответ: $S = 1/2\sqrt{271}$, $H(-\frac{175}{37}, \frac{84}{37}, \frac{20}{37}, \frac{422}{37}, \frac{454}{37})$.
8. $A(-1, 2, -9, 7, -6)$, $B(0, 6, -6, 10, -1)$, $C(4, 3, -7, 11, -5)$.
Ответ: $S = \sqrt{449}$, $H(\frac{113}{47}, \frac{126}{47}, -\frac{359}{47}, \frac{457}{47}, -\frac{250}{47})$.
9. $A(-10, 4, -1, 9, -10)$, $B(-9, 5, 0, 12, -9)$, $C(-7, 8, 0, 10, -6)$.
Ответ: $S = 1/2\sqrt{334}$, $H(-\frac{385}{43}, \frac{232}{43}, -\frac{28}{43}, \frac{402}{43}, -\frac{370}{43})$.
10. $A(-10, 1, -1, -9, 3)$, $B(-5, 3, 2, -6, 7)$, $C(-5, 5, 2, -4, 4)$.
Ответ: $S = 1/2\sqrt{1067}$, $H(-\frac{455}{76}, \frac{80}{19}, \frac{107}{76}, -\frac{379}{76}, \frac{289}{76})$.
11. $A(1, -4, -9, 3, -5)$, $B(4, -2, -5, 7, 0)$, $C(2, 0, -4, 5, 0)$.
Ответ: $S = 1/2\sqrt{874}$, $H(\frac{135}{71}, -\frac{28}{71}, -\frac{319}{71}, \frac{341}{71}, -\frac{35}{71})$.
12. $A(8, 2, -3, 2, -10)$, $B(9, 6, 2, 7, -6)$, $C(11, 3, 1, 3, -7)$.
Ответ: $S = \sqrt{263}$, $H(\frac{35}{3}, \frac{29}{9}, \frac{17}{9}, \frac{29}{9}, -\frac{19}{3})$.

13. $A(-1, 8, -2, -9, 5)$, $B(4, 11, 0, -6, 10)$, $C(4, 11, -1, -6, 7)$.

Ответ: $S = 1/2 \sqrt{431}$, $H(\frac{227}{48}, \frac{183}{16}, -\frac{41}{48}, -\frac{89}{16}, \frac{175}{24})$.

14. $A(-2, 8, 4, -1, -10)$, $B(-1, 12, 9, 3, -8)$, $C(1, 12, 7, 1, -7)$.

Ответ: $S = 1/2 \sqrt{610}$, $H(\frac{50}{47}, \frac{568}{47}, \frac{332}{47}, \frac{49}{47}, -\frac{326}{47})$.

15. $A(-6, -7, 6, 10, -1)$, $B(-3, -5, 9, 11, 2)$, $C(-3, -6, 11, 11, 1)$.

Ответ: $S = 1/2 \sqrt{191}$, $H(-\frac{141}{40}, -\frac{247}{40}, \frac{81}{8}, \frac{433}{40}, \frac{13}{20})$.

16. $A(-8, 2, 1, -8, -10)$, $B(-5, 4, 3, -5, -7)$, $C(-6, 7, 3, -5, -9)$.

Ответ: $S = 1/2 \sqrt{481}$, $H(-\frac{280}{43}, \frac{246}{43}, \frac{107}{43}, -\frac{248}{43}, -\frac{398}{43})$.

17. $A(-4, 9, -2, 3, 1)$, $B(-1, 10, 1, 6, 3)$, $C(-2, 12, 1, 5, 3)$.

Ответ: $S = 2 \sqrt{11}$, $H(-\frac{32}{15}, \frac{59}{5}, 4/5, \frac{73}{15}, \frac{43}{15})$.

18. $A(6, 6, 4, 10, 5)$, $B(10, 7, 6, 13, 8)$, $C(8, 8, 8, 11, 9)$.

Ответ: $S = 1/2 \sqrt{510}$, $H(\frac{312}{41}, \frac{312}{41}, \frac{296}{41}, \frac{443}{41}, \frac{337}{41})$.

19. $A(-9, 6, 10, -2, 2)$, $B(-6, 7, 11, 0, 3)$, $C(-4, 7, 15, 0, 7)$.

Ответ: $S = \sqrt{95}$, $H(-\frac{57}{8}, \frac{51}{8}, \frac{95}{8}, -5/4, \frac{31}{8})$.

20. $A(0, -5, 4, -10, -5)$, $B(1, 0, 6, -8, -4)$, $C(5, -2, 7, -7, -3)$.

Ответ: $S = \sqrt{201}$, $H(\frac{85}{28}, -\frac{89}{28}, \frac{163}{28}, -\frac{229}{28}, -\frac{53}{14})$.

21. $A(-8, -3, 8, 1, 10)$, $B(-3, -1, 10, 3, 11)$, $C(-4, 2, 11, 2, 13)$.

Ответ: $S = 1/2 \sqrt{599}$, $H(-\frac{79}{15}, \frac{5}{12}, \frac{201}{20}, \frac{101}{60}, \frac{241}{20})$.

22. $A(-6, 9, -4, -5, -4)$, $B(-5, 11, -2, -3, 0)$, $C(-2, 10, 0, -4, 1)$.

Ответ: $S = 1/2 \sqrt{415}$, $H(-\frac{210}{59}, \frac{567}{59}, -\frac{92}{59}, -\frac{259}{59}, -\frac{56}{59})$.

23. $A(3, 1, -9, -1, 6)$, $B(6, 4, -5, 4, 9)$, $C(4, 6, -7, 0, 9)$.

Ответ: $S = 2 \sqrt{70}$, $H(4, 6, -7, 0, 9)$.

24. $A(8, -6, -1, -3, 4)$, $B(12, -5, 1, 2, 7)$, $C(10, -4, 2, -2, 6)$.

Ответ: $S = 1/2 \sqrt{481}$, $H(\frac{115}{11}, -\frac{39}{11}, \frac{59}{22}, -\frac{39}{22}, \frac{71}{11})$.

25. $A(3, 8, -6, 1, 10)$, $B(8, 12, -3, 6, 12)$, $C(4, 11, -4, 6, 11)$.

Ответ: $S = \sqrt{165}$, $H(\frac{17}{4}, \frac{47}{4}, -7/2, \frac{29}{4}, \frac{45}{4})$.

26. $A(8, -2, -6, -1, 4)$, $B(11, -1, -3, 4, 6)$, $C(12, 2, -4, 0, 5)$.

Ответ: $S = 1/2 \sqrt{983}$, $H(\frac{210}{19}, \frac{20}{19}, -\frac{85}{19}, -\frac{9}{38}, \frac{181}{38})$.

27. $A(-2, 4, 2, -8, -9)$, $B(0, 9, 5, -7, -7)$, $C(-1, 6, 6, -3, -4)$.

Ответ: $S = \sqrt{383}$, $H(-\frac{103}{71}, \frac{362}{71}, \frac{298}{71}, -\frac{373}{71}, -\frac{444}{71})$.

28. $A(-8, -6, 0, -8, -6)$, $B(-4, -5, 5, -4, -2)$, $C(-5, -4, 4, -5, -2)$.

Ответ: $S = \sqrt{38}$, $H(-\frac{41}{9}, -\frac{100}{27}, \frac{124}{27}, -\frac{41}{9}, -\frac{38}{27})$.

29. $A(-4, -7, 9, 1, -3)$, $B(-3, -4, 14, 6, 1)$, $C(1, -3, 10, 6, -1)$.

Ответ: $S = 1/2 \sqrt{2371}$, $H(-\frac{9}{71}, -\frac{277}{71}, \frac{694}{71}, \frac{346}{71}, -\frac{103}{71})$.

30. $A(-4, -8, -10, 8, 2)$, $B(0, -5, -5, 10, 5)$, $C(-1, -3, -5, 11, 6)$.

Ответ: $S = 7\sqrt{2}$, $H(-3/2, -\frac{23}{6}, -\frac{35}{6}, 21/2, 16/3)$.

Задача 2.5. Найти расстояние от точки M :

a) до гиперплоскости \mathcal{P}_1 ;

b) до плоскости \mathcal{P}_2 , проходящей через точку A в направлении подпространства $\langle \vec{v}_1, \vec{v}_2 \rangle$.

1. $M(-7, -4, -3, -7)$, $\mathcal{P}_1: 5x^1 - 11x^2 - 5x^3 + 5x^4 - 17 = 0$;
 $\mathcal{P}_2: A(2, 2, -3, 0)$, $\vec{v}_1 = (-2, 0, 5, 5)$, $\vec{v}_2 = (-6, -5, 0, 5)$.

Ответ: $d_1 = 2$, $d_2 = 10$

2. $M(13, -1, -5, -1)$, $\mathcal{P}_1: 9x^1 + 4x^2 - 8x^3 - 8x^4 - 146 = 0$;
 $\mathcal{P}_2: A(-3, 0, 1, -3)$, $\vec{v}_1 = (3, 4, 0, 4)$, $\vec{v}_2 = (-9, 4, -4, 0)$.

Ответ: $d_1 = 1$, $d_2 = 11$

3. $M(-3, -1, -2, -3)$, $\mathcal{P}_1: 8x^1 + 8x^2 + 4x^3 + 5x^4 + 29 = 0$;
 $\mathcal{P}_2: A(-2, -2, 2, 0)$, $\vec{v}_1 = (-6, 0, 1, 1)$, $\vec{v}_2 = (-6, -1, 1, 0)$.

Ответ: $d_1 = 2$, $d_2 = 5$

4. $M(10, -5, 1, 12)$, $\mathcal{P}_1: 5x^1 - 5x^2 - 11x^3 + 5x^4 - 96 = 0$;
 $\mathcal{P}_2: A(-3, -3, 2, 3)$, $\vec{v}_1 = (0, 7, 0, 7)$, $\vec{v}_2 = (-6, 7, -7, 0)$.

Ответ: $d_1 = 2$, $d_2 = 11$

5. $M(3, 0, -11, 12)$, $\mathcal{P}_1: 6x^1 + 9x^2 + 4x^3 + 6x^4 - 33 = 0$;
 $\mathcal{P}_2: A(-2, 0, 0, 2)$, $\vec{v}_1 = (16, 5, 5, 0)$, $\vec{v}_2 = (16, 0, 5, -5)$.

Ответ: $d_1 = 1$, $d_2 = 14$

6. $M(-2, 10, -13, -20)$, $\mathcal{P}_1: x^1 + 7x^2 + 5x^3 - 5x^4 - 83 = 0$;
 $\mathcal{P}_2: A(-3, -2, -1, -1)$, $\vec{v}_1 = (4, 0, 9, 9)$, $\vec{v}_2 = (4, -9, 0, 9)$.

Ответ: $d_1 = 2$, $d_2 = 10$

7. $M(-4, 8, 5, -14)$, $\mathcal{P}_1: 8x^1 + 10x^2 - 10x^3 - 5x^4 - 51 = 0$;
 $\mathcal{P}_2: A(1, 3, -3, 1)$, $\vec{v}_1 = (18, 5, 5, 0)$, $\vec{v}_2 = (18, 0, 5, -5)$.

Ответ: $d_1 = 1$, $d_2 = 17$

8. $M(8, 18, 7, 9)$, $\mathcal{P}_1: 6x^1 - 8x^2 + 10x^3 + 5x^4 + 11 = 0$;
 $\mathcal{P}_2: A(1, 2, -3, 2)$, $\vec{v}_1 = (-12, 11, 0, 11)$, $\vec{v}_2 = (-8, 0, -11, 11)$.

Ответ: $d_1 = 2$, $d_2 = 14$

9. $M(36, 4, 3, -1)$, $\mathcal{P}_1: 7x^1 - 2x^2 + 8x^3 + 2x^4 - 255 = 0$;
 $\mathcal{P}_2: A(2, 3, 1, 1)$, $\vec{v}_1 = (-15, 4, 4, 0)$, $\vec{v}_2 = (-15, 4, 0, -4)$.

Ответ: $d_1 = 1$, $d_2 = 13$

10. $M(0, -6, 12, 9)$, $\mathcal{P}_1: 4x^1 + 2x^2 - 5x^3 - 2x^4 + 76 = 0$;
 $\mathcal{P}_2: A(3, 3, 2, 3)$, $\vec{v}_1 = (-4, 0, 4, 4)$, $\vec{v}_2 = (-3, -4, 0, 4)$.

Ответ: $d_1 = 2$, $d_2 = 9$

11. $M(26, 5, -7, 5)$, $\mathcal{P}_1: 7x^1 + x^2 - 5x^3 + 5x^4 - 237 = 0$;
 $\mathcal{P}_2: A(2, 1, -2, 3)$, $\vec{v}_1 = (-3, 0, 6, 6)$, $\vec{v}_2 = (15, -6, 6, 0)$.

Ответ: $d_1 = 1$, $d_2 = 15$

12. $M(21, -4, -6, -10)$, $\mathcal{P}_1: 4x^1 + 10x^2 - 2x^3 + 7x^4 - 12 = 0$;
 $\mathcal{P}_2: A(-2, 0, 1, -3)$, $\vec{v}_1 = (14, 3, 0, 3)$, $\vec{v}_2 = (-6, 3, -3, 0)$.

Ответ: $d_1 = 2$, $d_2 = 15$

13. $M(-14, 7, -6, -9)$, $\mathcal{P}_1: 11x^1 - x^2 + 11x^3 + 9x^4 + 272 = 0$;
 $\mathcal{P}_2: A(-1, -3, -3, -3)$, $\vec{v}_1 = (-2, 2, 0, 2)$, $\vec{v}_2 = (-9, 0, -2, 2)$.

Ответ: $d_1 = 2$, $d_2 = 13$

14. $M(-2, 5, -6, 12)$, $\mathcal{P}_1: 7x^1 - 8x^2 + 2x^3 + 2x^4 + 20 = 0$;
 $\mathcal{P}_2: A(-3, 3, 0, 3)$, $\vec{v}_1 = (8, 4, 4, 0)$, $\vec{v}_2 = (11, 0, 4, -4)$.

Ответ: $d_1 = 2$, $d_2 = 9$

15. $M(0, -13, -3, -8)$, $\mathcal{P}_1 : 4x^1 - 8x^2 - 8x^3 + 9x^4 - 26 = 0$;
 $\mathcal{P}_2 : A(3, -1, 3, -3)$, $\vec{v}_1 = (-4, 0, 7, 7)$, $\vec{v}_2 = (0, -7, 0, 7)$.
Ответ: $d_1 = 2$, $d_2 = 10$
16. $M(-15, -10, -1, 13)$, $\mathcal{P}_1 : x^1 - 9x^2 - 3x^3 + 3x^4 - 107 = 0$;
 $\mathcal{P}_2 : A(-2, 2, 2, -3)$, $\vec{v}_1 = (9, 8, 8, 0)$, $\vec{v}_2 = (-4, 8, 0, -8)$.
Ответ: $d_1 = 1$, $d_2 = 13$
17. $M(5, -16, -8, 9)$, $\mathcal{P}_1 : x^1 - x^2 - 7x^3 - 7x^4 - 4 = 0$;
 $\mathcal{P}_2 : A(-2, 3, -2, -3)$, $\vec{v}_1 = (-4, 7, 7, 0)$, $\vec{v}_2 = (-10, 7, 0, -7)$.
Ответ: $d_1 = 1$, $d_2 = 10$
18. $M(10, 11, -7, 3)$, $\mathcal{P}_1 : 3x^1 - 3x^2 + x^3 + 9x^4 - 7 = 0$;
 $\mathcal{P}_2 : A(1, 3, -2, 3)$, $\vec{v}_1 = (0, 1, 1, 0)$, $\vec{v}_2 = (8, 0, 1, -1)$.
Ответ: $d_1 = 1$, $d_2 = 10$
19. $M(31, 3, 1, 1)$, $\mathcal{P}_1 : 10x^1 + 5x^2 - 10x^3 + 8x^4 - 306 = 0$;
 $\mathcal{P}_2 : A(0, 3, 2, -1)$, $\vec{v}_1 = (12, 11, 11, 0)$, $\vec{v}_2 = (-8, 0, 11, -11)$.
Ответ: $d_1 = 1$, $d_2 = 18$
20. $M(-12, -1, 4, 14)$, $\mathcal{P}_1 : 7x^1 - 2x^2 + 10x^3 + 4x^4 - 1 = 0$;
 $\mathcal{P}_2 : A(-1, 3, -3, -1)$, $\vec{v}_1 = (-8, 11, 11, 0)$, $\vec{v}_2 = (-8, 11, 0, -11)$.
Ответ: $d_1 = 1$, $d_2 = 13$
21. $M(16, 1, 6, -3)$, $\mathcal{P}_1 : 3x^1 + x^2 + 9x^3 + 3x^4 - 84 = 0$;
 $\mathcal{P}_2 : A(-3, 3, -2, -2)$, $\vec{v}_1 = (6, 1, 0, 1)$, $\vec{v}_2 = (12, 0, -1, 1)$.
Ответ: $d_1 = 1$, $d_2 = 10$
22. $M(10, -20, -20, 8)$, $\mathcal{P}_1 : 3x^1 - x^2 - 3x^3 + 9x^4 - 162 = 0$;
 $\mathcal{P}_2 : A(2, -2, 0, 3)$, $\vec{v}_1 = (-3, 10, 0, 10)$, $\vec{v}_2 = (15, 0, -10, 10)$.
Ответ: $d_1 = 2$, $d_2 = 17$
23. $M(11, 7, 3, -10)$, $\mathcal{P}_1 : 9x^1 - 3x^2 - x^3 + 3x^4 - 35 = 0$;
 $\mathcal{P}_2 : A(0, 0, -3, 0)$, $\vec{v}_1 = (12, 0, 8, 8)$, $\vec{v}_2 = (9, -8, 0, 8)$.
Ответ: $d_1 = 1$, $d_2 = 13$
24. $M(-5, -6, 13, 5)$, $\mathcal{P}_1 : 8x^1 + 4x^2 + 9x^3 - 8x^4 + 17 = 0$;
 $\mathcal{P}_2 : A(2, 3, -1, -1)$, $\vec{v}_1 = (-4, 9, 9, 0)$, $\vec{v}_2 = (-6, 9, 0, -9)$.
Ответ: $d_1 = 2$, $d_2 = 14$

25. $M(-20, 10, -5, -10)$, $\mathcal{P}_1: 4x^1 + 6x^2 - 6x^3 + 9x^4 + 67 = 0$;
 $\mathcal{P}_2: A(1, 2, -3, -1)$, $\vec{v}_1 = (8, 0, 1, 1)$, $\vec{v}_2 = (14, -1, 0, 1)$.

Ответ: $d_1 = 1$, $d_2 = 10$

26. $M(11, 13, -6, 18)$, $\mathcal{P}_1: 8x^1 - 3x^2 + 2x^3 + 2x^4 - 64 = 0$;
 $\mathcal{P}_2: A(-1, -1, -1, 2)$, $\vec{v}_1 = (3, 6, 6, 0)$, $\vec{v}_2 = (-15, 0, 6, -6)$.

Ответ: $d_1 = 1$, $d_2 = 15$

27. $M(6, 10, 13, 5)$, $\mathcal{P}_1: 11x^1 - 9x^2 - x^3 - 11x^4 + 74 = 0$;
 $\mathcal{P}_2: A(2, -2, -3, 0)$, $\vec{v}_1 = (13, 0, 8, 8)$, $\vec{v}_2 = (1, -8, 0, 8)$.

Ответ: $d_1 = 1$, $d_2 = 13$

28. $M(18, -6, 8, 9)$, $\mathcal{P}_1: 4x^1 + 6x^2 - 9x^3 + 6x^4 - 5 = 0$;
 $\mathcal{P}_2: A(1, 2, 1, 1)$, $\vec{v}_1 = (9, 0, 2, 2)$, $\vec{v}_2 = (10, -2, 0, 2)$.

Ответ: $d_1 = 1$, $d_2 = 9$

29. $M(4, 11, 3, -9)$, $\mathcal{P}_1: 4x^1 - x^2 + 10x^3 - 2x^4 - 42 = 0$;
 $\mathcal{P}_2: A(1, -3, 2, 1)$, $\vec{v}_1 = (0, 4, 0, 4)$, $\vec{v}_2 = (-7, 0, -4, 4)$.

Ответ: $d_1 = 1$, $d_2 = 15$

30. $M(21, 1, 3, -2)$, $\mathcal{P}_1: 6x^1 - 5x^2 + 4x^3 + 2x^4 - 120 = 0$;
 $\mathcal{P}_2: A(3, 3, 3, -3)$, $\vec{v}_1 = (-7, 0, 2, 2)$, $\vec{v}_2 = (-9, -2, 0, 2)$.

Ответ: $d_1 = 1$, $d_2 = 7$

Задача 2.6. Найти расстояние между прямой ℓ , проходящей через точки A и B , и плоскостью \mathcal{P} , проходящей через точку M параллельно подпространству $\mathbf{L} = \langle \vec{v}_1, \vec{v}_2 \rangle$.

1. $A(-15, -2, -2, 13)$, $B(-22, -4, 0, 11)$;
 $M(-1, 0, -2, -3)$, $\vec{v}_1 = (-3, -2, -2, -2)$, $\vec{v}_2 = (-13, 2, 2, -2)$.

Ответ: $d = 18$

2. $A(11, -13, 4, -5)$, $B(17, -9, 0, -9)$;
 $M(-1, -3, 0, -1)$, $\vec{v}_1 = (7, 2, 2, 2)$, $\vec{v}_2 = (1, -2, -2, 2)$.

Ответ: $d = 14$

3. $A(-16, -1, 1, -2)$, $B(-6, 7, -7, 6)$;
 $M(-2, -3, -1, 1)$, $\vec{v}_1 = (9, 4, 4, 4)$, $\vec{v}_2 = (1, -4, -4, 4)$.

Ответ: $d = 7$

4. $A(3, -7, 1, -6)$, $B(5, -3, -3, -10)$;
 $M(-3, -3, 3, -1)$, $\vec{v}_1 = (-7, -2, -2, -2)$, $\vec{v}_2 = (11, 2, -2, 2)$.

Ответ: $d = 7$

5. $A(-5, -8, 24, -9)$, $B(-3, -24, 8, 7)$;
 $M(-3, 2, 3, -3)$, $\vec{v}_1 = (11, 8, 8, 8)$, $\vec{v}_2 = (-21, -8, 8, -8)$.

Ответ: $d = 15$

6. $A(-12, 29, 17, 16)$, $B(-9, 34, 12, 11)$;
 $M(2, 1, -1, 0)$, $\vec{v}_1 = (-21, 5, 5, 5)$, $\vec{v}_2 = (3, -5, 5, -5)$.

Ответ: $d = 28$

7. $A(-10, -6, -4, -3)$, $B(-14, -5, -3, -4)$;
 $M(-1, -2, -2, 1)$, $\vec{v}_1 = (-12, 1, 1, 1)$, $\vec{v}_2 = (4, -1, 1, -1)$.

Ответ: $d = 7$

8. $A(-8, 8, 6, -4)$, $B(-19, 14, 0, 2)$;
 $M(2, -2, 2, 1)$, $\vec{v}_1 = (3, -6, -6, -6)$, $\vec{v}_2 = (1, -6, 6, 6)$.

Ответ: $d = 9$

9. $A(-7, 7, -5, 4)$, $B(8, 11, -9, 8)$;
 $M(1, -3, 2, -2)$, $\vec{v}_1 = (-1, -4, -4, -4)$, $\vec{v}_2 = (5, 4, 4, -4)$.

Ответ: $d = 13$

10. $A(10, 2, -5, 5)$, $B(9, 1, -4, 6)$;
 $M(-3, 3, -2, 2)$, $\vec{v}_1 = (3, -1, -1, -1)$, $\vec{v}_2 = (-9, 1, -1, 1)$.

Ответ: $d = 6$

11. $A(6, 3, 9, -19)$, $B(-8, 11, 1, -27)$;
 $M(2, 3, 0, -3)$, $\vec{v}_1 = (-9, 4, 4, 4)$, $\vec{v}_2 = (-9, -4, 4, -4)$.

Ответ: $d = 15$

12. $A(13, 9, 15, -33)$, $B(20, 19, 5, -23)$;
 $M(3, -3, -1, -3)$, $\vec{v}_1 = (-9, 10, 10, 10)$, $\vec{v}_2 = (19, -10, -10, 10)$.

Ответ: $d = 30$

13. $A(59, 7, -3, 1)$, $B(62, -4, 8, 12)$;
 $M(-3, 3, -1, 1)$, $\vec{v}_1 = (-21, -11, -11, -11)$, $\vec{v}_2 = (19, 11, -11, 11)$.

Ответ: $d = 36$

14. $A(32, 12, 1, -3)$, $B(39, 9, 4, 0)$;
 $M(2, 0, 3, -3)$, $\vec{v}_1 = (11, -3, -3, -3)$, $\vec{v}_2 = (-13, 3, -3, 3)$.
Ответ: $d = 20$
15. $A(12, 22, -1, -25)$, $B(10, 12, 9, -35)$;
 $M(2, 2, -1, -3)$, $\vec{v}_1 = (-11, -5, -5, -5)$, $\vec{v}_2 = (-11, 5, 5, -5)$.
Ответ: $d = 28$
16. $A(-3, 3, 10, -5)$, $B(5, 13, 0, -15)$;
 $M(-2, -1, -2, -3)$, $\vec{v}_1 = (4, 5, 5, 5)$, $\vec{v}_2 = (0, -5, 5, -5)$.
Ответ: $d = 7$
17. $A(-9, -2, -18, -13)$, $B(-39, 0, -20, -15)$;
 $M = (1, -2, -2, 1)$, $\vec{v}_1 = (-13, -1, -1, -1)$, $\vec{v}_2 = (-1, 1, 1, -1)$.
Ответ: $d = 20$
18. $A(16, 1, -14, -9)$, $B(26, -13, 0, 5)$;
 $M(0, 3, 0, 3)$, $\vec{v}_1 = (-7, -7, -7, -7)$, $\vec{v}_2 = (9, 7, 7, -7)$.
Ответ: $d = 20$
19. $A(-22, -10, -5, 1)$, $B(-22, -5, -10, 6)$;
 $M(-1, -2, -1, -3)$, $\vec{v}_1 = (12, 5, 5, 5)$, $\vec{v}_2 = (-4, -5, -5, 5)$.
Ответ: $d = 9$
20. $A(-11, -6, -6, -7)$, $B(-29, 2, 2, -15)$;
 $M(-1, 3, -2, -3)$, $\vec{v}_1 = (-1, 4, 4, 4)$, $\vec{v}_2 = (-7, -4, 4, -4)$.
Ответ: $d = 7$
21. $A(-6, 7, -21, 35)$, $B(-56, 23, -37, 51)$;
 $M(-2, -3, -1, -1)$, $\vec{v}_1 = (-5, 8, 8, 8)$, $\vec{v}_2 = (-15, -8, -8, 8)$.
Ответ: $d = 34$
22. $A(5, 9, -9, 36)$, $B(12, 2, -2, 43)$;
 $M(-1, -3, 1, 0)$, $\vec{v}_1 = (5, 7, 7, 7)$, $\vec{v}_2 = (-15, -7, 7, -7)$.
Ответ: $d = 28$
23. $A(21, -4, -4, 4)$, $B(17, -7, -1, 7)$;
 $M(-2, 0, -2, 2)$, $\vec{v}_1 = (8, -3, -3, -3)$, $\vec{v}_2 = (-12, 3, -3, 3)$.
Ответ: $d = 9$

24. $A(24, 4, -1, 7)$, $B(7, 0, 3, 11)$;
 $M(0, 1, -3, -3)$, $\vec{v}_1 = (-23, 4, 4, 4)$, $\vec{v}_2 = (3, -4, 4, -4)$.

Ответ: $d = 15$

25. $A(4, -9, 9, 1)$, $B(24, -8, 8, 2)$;
 $M(-1, -1, -1, 3)$, $\vec{v}_1 = (4, 1, 1, 1)$, $\vec{v}_2 = (0, -1, -1, 1)$.

Ответ: $d = 13$

26. $A(-1, 7, -8, 15)$, $B(-39, 15, -16, 23)$;
 $M(3, 3, 3, 3)$, $\vec{v}_1 = (-3, -4, -4, -4)$, $\vec{v}_2 = (11, 4, 4, -4)$.

Ответ: $d = 13$

27. $A(1, -6, 0, -7)$, $B(4, -5, -1, -8)$;
 $M(0, -3, 3, -2)$, $\vec{v}_1 = (-9, 1, 1, 1)$, $\vec{v}_2 = (7, -1, 1, -1)$.

Ответ: $d = 6$

28. $A(-23, -9, -15, 19)$, $B(-13, -15, -9, 25)$;
 $M(1, 1, -3, 1)$, $\vec{v}_1 = (-5, 3, 3, 3)$, $\vec{v}_2 = (-13, -3, 3, -3)$.

Ответ: $d = 28$

29. $A(-21, 2, 1, -5)$, $B(-30, -5, 8, -12)$;
 $M(0, 1, 2, 2)$, $\vec{v}_1 = (-7, -7, -7, -7)$, $\vec{v}_2 = (-7, 7, 7, -7)$.

Ответ: $d = 10$

30. $A(2, -8, -3, 9)$, $B(0, 0, -11, 17)$;
 $M(2, -2, 2, -1)$, $\vec{v}_1 = (9, 4, 4, 4)$, $\vec{v}_2 = (-13, -4, -4, 4)$.

Ответ: $d = 9$

Задача 2.7. В четырехмерном пространстве заданы точки O , A , B , C и D . Найти:

- i. Объем трехмерного симплекса $ABCD$;
- ii. Угол наклона ребра AD симплекса $ABCD$ к плоскости основания ABC ;
- iii. Объем четырехмерного параллелепипеда, построенного на векторах \vec{OA} , \vec{OB} , \vec{OC} , \vec{OD} ;
- iv. Объем четырехмерного симплекса $OABCD$.

1. $O(6, 11, 8, 12)$, $A(2, -2, -4, -4)$, $B(4, -2, -4, -2)$,
 $C(3, 3, 1, -3)$, $D(9, -3, 3, -5)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{80}{3}$, $V_1 = 880$

2. $O(10, 6, 8, 7)$, $A(-2, 2, 2, 6)$, $B(4, 2, 2, 12)$,
 $C(0, 6, 6, 8)$, $D(5, 3, 9, 7)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 48$, $V_1 = 1296$

3. $O(7, 12, 11, 9)$, $A(-4, 4, 4, 4)$, $B(0, 2, 2, 8)$,
 $C(-5, 7, 7, 3)$, $D(3, 5, 11, 5)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 20$, $V_1 = 420$

4. $O(9, 7, 12, 6)$, $A(0, -2, 6, 0)$, $B(3, -3, 5, 3)$,
 $C(-1, 1, 9, -1)$, $D(7, -3, 13, -1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{64}{3}$, $V_1 = 384$

5. $O(12, 12, 11, 11)$, $A(-5, 3, -1, -1)$, $B(-3, 3, -1, 1)$,
 $C(-6, 6, 2, -2)$, $D(2, 4, 6, 0)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 12$, $V_1 = 72$

6. $O(9, 8, 12, 6)$, $A(-2, -4, -2, 0)$, $B(3, -5, -3, 5)$,
 $C(-4, 0, 2, -2)$, $D(5, -5, 5, -1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 48$, $V_1 = 432$

7. $O(9, 6, 11, 12)$, $A(-2, -4, 0, 2)$, $B(1, -5, -1, 5)$,
 $C(-2, -2, 2, 2)$, $D(5, -3, 7, 3)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 12$, $V_1 = 0$

8. $O(12, 7, 9, 11)$, $A(-3, 3, -3, -5)$, $B(1, 5, -1, -1)$,
 $C(-5, 7, 1, -7)$, $D(4, 2, 4, -6)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{160}{3}$, $V_1 = 1440$

9. $O(8, 10, 8, 11)$, $A(-3, 3, -1, 1)$, $B(2, 2, -2, 6)$,
 $C(-5, 7, 3, -1)$, $D(4, 2, 6, 0)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 48$, $V_1 = 144$

10. $O(11, 12, 11, 8)$, $A(3, -3, 1, 3)$, $B(7, -3, 1, 7)$,
 $C(4, 0, 4, 4)$, $D(10, -2, 8, 4)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 24$, $V_1 = 576$

11. $O(8, 12, 8, 10)$, $A(-1, 3, -1, -5)$, $B(3, 5, 1, -1)$,
 $C(1, 7, 3, -3)$, $D(6, 4, 6, -4)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 24$, $V_1 = 432$

12. $O(6, 8, 10, 11)$, $A(2, 2, -6, 2)$, $B(4, 2, -6, 4)$,
 $C(0, 6, -2, 0)$, $D(9, 1, 1, 1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{64}{3}$, $V_1 = 960$

13. $O(6, 8, 9, 9)$, $A(-4, -2, 2, 0)$, $B(1, -1, 3, 5)$,
 $C(-6, 2, 6, -2)$, $D(3, -1, 9, 1)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 44$, $V_1 = 528$

14. $O(8, 11, 10, 12)$, $A(1, -1, 1, -5)$, $B(6, 0, 2, 0)$,
 $C(2, 4, 6, -4)$, $D(8, 0, 8, -4)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 48$, $V_1 = 1008$

15. $O(12, 9, 9, 11)$, $A(0, 0, 2, 2)$, $B(5, -1, 1, 7)$,
 $C(-1, 5, 7, 1)$, $D(7, -1, 9, 1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 64$, $V_1 = 960$

16. $O(7, 12, 6, 12)$, $A(-5, -1, 1, -3)$, $B(-3, -1, 1, -1)$,
 $C(-6, 4, 6, -4)$, $D(2, 0, 8, -2)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 20$, $V_1 = 300$

17. $O(9, 8, 9, 11)$, $A(-1, -7, 1, -1)$, $B(3, -7, 1, 3)$,
 $C(0, -2, 6, 0)$, $D(6, -6, 8, 0)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 40$, $V_1 = 600$

18. $O(7, 12, 11, 10)$, $A(0, 4, 2, 2)$, $B(3, 3, 1, 5)$,
 $C(-1, 7, 5, 1)$, $D(7, 3, 9, 1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{64}{3}$, $V_1 = 128$

19. $O(6, 8, 11, 9)$, $A(0, -2, 0, 2)$, $B(3, -1, 1, 5)$,
 $C(-1, 3, 5, 1)$, $D(7, -3, 7, 1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{128}{3}$, $V_1 = 256$

20. $O(8, 8, 7, 10)$, $A(-2, -2, -4, 0)$, $B(1, -1, -3, 3)$,
 $C(-2, 0, -2, 0)$, $D(5, -3, 3, -1)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 16$, $V_1 = 48$

21. $O(9, 11, 10, 10)$, $A(3, 3, -1, -1)$, $B(8, 2, -2, 4)$,
 $C(2, 8, 4, -2)$, $D(10, 2, 6, -2)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 64$, $V_1 = 1536$

22. $O(8, 8, 7, 8)$, $A(1, 3, -3, 3)$, $B(4, 4, -2, 6)$,
 $C(2, 6, 0, 4)$, $D(8, 2, 4, 2)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{64}{3}$, $V_1 = 192$

23. $O(6, 10, 8, 6)$, $A(3, 3, 1, -3)$, $B(8, 4, 2, 2)$,
 $C(5, 7, 5, -1)$, $D(10, 2, 8, -4)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 48$, $V_1 = 864$

24. $O(7, 8, 12, 11)$, $A(5, 3, -3, 3)$, $B(9, 3, -3, 7)$,
 $C(3, 7, 1, 1)$, $D(12, 2, 4, 2)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{128}{3}$, $V_1 = 2048$

25. $O(11, 7, 7, 9)$, $A(4, 0, -2, 2)$, $B(8, 2, 0, 6)$,
 $C(4, 4, 2, 2)$, $D(11, 1, 5, 3)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 32$, $V_1 = 192$

26. $O(8, 9, 7, 8)$, $A(0, 0, 8, 0)$, $B(4, -2, 6, 4)$,
 $C(-2, 4, 12, -2)$, $D(7, 1, 15, 1)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 24$, $V_1 = 720$

27. $O(9, 8, 12, 10)$, $A(5, -3, -1, -1)$, $B(7, -3, -1, 1)$,
 $C(3, 1, 3, -3)$, $D(12, -4, 6, -2)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{64}{3}$, $V_1 = 576$

28. $O(9, 6, 10, 7)$, $A(-2, 0, 0, -2)$, $B(4, 0, 0, 4)$,
 $C(-2, 6, 6, -2)$, $D(5, -1, 7, -3)$.

ОТВЕТ: $\arcsin(4/5)$, $V = 96$, $V_1 = 576$

29. $O(6, 9, 10, 8)$, $A(-1, -1, 1, -3)$, $B(3, 1, 3, 1)$,
 $C(-1, 1, 3, -3)$, $D(6, 0, 8, -2)$.

ОТВЕТ: $\arcsin(3/5)$, $V = 16$, $V_1 = 144$

30. $O(8, 10, 12, 6)$, $A(-1, -1, 1, -3)$, $B(2, -2, 0, 0)$,
 $C(0, 4, 6, -2)$, $D(6, -2, 8, -4)$.

ОТВЕТ: $\arcsin(4/5)$, $V = \frac{128}{3}$, $V_1 = 0$

3. Аффинные преобразования

Задача 3.1. Найти образ точки M при гомотетии с центром в точке C и коэффициентом k .

1. $M(-8, 2, -8), C(8, -4, -2), k = 2/3$.

Ответ: $M'(-8/3, 0, -6)$

2. $M(1, -10, 6), C(8, -7, -1), k = 1/2$.

Ответ: $M'(9/2, -17/2, 5/2)$

3. $M(7, 3, 10), C(8, 5, -10), k = 7/6$.

Ответ: $M'(\frac{41}{6}, 8/3, \frac{40}{3})$

4. $M(-5, 0, -4), C(-6, -9, -3), k = 5/3$.

Ответ: $M'(-13/3, 6, -14/3)$

5. $M(-1, -4, 6), C(-5, -9, 1), k = 1/2$.

Ответ: $M'(-3, -13/2, 7/2)$

6. $M(-10, 8, -10), C(9, 7, 5), k = 7/6$.

Ответ: $M'(-\frac{79}{6}, \frac{49}{6}, -\frac{25}{2})$

7. $M(7, 3, 6), C(-8, 8, -1), k = 5/3$.

Ответ: $M'(17, -1/3, \frac{32}{3})$

8. $M(7, -1, -9), C(-7, 5, -10), k = 1/2$.

Ответ: $M'(0, 2, -19/2)$

9. $M(10, -7, 9), C(-4, -8, 8), k = 2/3$.

Ответ: $M'(16/3, -\frac{22}{3}, \frac{26}{3})$

10. $M(-5, -3, -7), C(5, 7, 10), k = 5/6$.

Ответ: $M'(-10/3, -4/3, -\frac{25}{6})$

11. $M(3, -3, 8), C(-5, 8, -4), k = 2/3$.

Ответ: $M'(1/3, 2/3, 4)$

12. $M(-3, 6, 0), C(9, -5, 0), k = 5/6$.

Ответ: $M'(-1, \frac{25}{6}, 0)$

13. $M(6, -10, 7), C(-1, -10, 3), k = 1/3.$

Ответ: $M'(4/3, -10, 13/3)$

14. $M(0, -8, -8), C(-5, 0, -3), k = 2/3.$

Ответ: $M'(-5/3, -16/3, -\frac{19}{3})$

15. $M(2, 4, -10), C(8, 7, -2), k = 5/3.$

Ответ: $M'(-2, 2, -\frac{46}{3})$

16. $M(-4, 10, 8), C(4, 0, 5), k = 1/3.$

Ответ: $M'(4/3, 10/3, 6)$

17. $M(9, -3, 5), C(3, -1, -4), k = 1/6.$

Ответ: $M'(4, -4/3, -5/2)$

18. $M(-2, 3, 3), C(-5, -5, -2), k = 4/3.$

Ответ: $M'(-1, \frac{17}{3}, 14/3)$

19. $M(5, -5, 5), C(-9, -1, 2), k = 1/2.$

Ответ: $M'(-2, -3, 7/2)$

20. $M(-2, -10, -8), C(-10, 3, 9), k = 1/3.$

Ответ: $M'(-\frac{22}{3}, -4/3, 10/3)$

21. $M(-1, 0, -4), C(-8, -9, -2), k = 1/6.$

Ответ: $M'(-\frac{41}{6}, -15/2, -7/3)$

22. $M(-1, 1, 6), C(-10, -8, 0), k = 2/3.$

Ответ: $M'(-4, -2, 4)$

23. $M(2, 3, 8), C(6, 6, 6), k = 1/3.$

Ответ: $M'(14/3, 5, \frac{20}{3})$

24. $M(-3, -6, -5), C(7, 6, -7), k = 3/2.$

Ответ: $M'(-8, -12, -4)$

25. $M(-7, 2, 0), C(8, -4, -2), k = 2/3.$

Ответ: $M'(-2, 0, -2/3)$

26. $M(-3, 9, -4), C(-7, -10, 1), k = 2/3.$

Ответ: $M'(-13/3, 8/3, -7/3)$

27. $M(4, 2, 1)$, $C(-3, 3, -3)$, $k = 4/3$.

Ответ: $M'(\frac{19}{3}, 5/3, 7/3)$

28. $M(7, -5, -5)$, $C(-3, 6, 0)$, $k = 1/6$.

Ответ: $M'(-4/3, \frac{25}{6}, -5/6)$

Задача 3.2. Найти аффинное преобразование пространства, являющееся сжатием к плоскости \mathcal{P} с коэффициентом k . Система координат прямоугольная.

1. $\mathcal{P} : 3x - 4y - z + 3 = 0$, $k = 4$.

Ответ: $[x' = \frac{77}{104}x + \frac{9}{26}y + \frac{9}{104}z - \frac{27}{104}, y' = \frac{9}{26}x + \frac{7}{13}y - \frac{3}{26}z + \frac{9}{26}, z' = \frac{9}{104}x - \frac{3}{26}y + \frac{101}{104}z + \frac{9}{104}]$

2. $\mathcal{P} : 5x + 5y - 2z - 4 = 0$, $k = 3$.

Ответ: $[x' = \frac{56}{81}x - \frac{25}{81}y + \frac{10}{81}z + \frac{20}{81}, y' = -\frac{25}{81}x + \frac{56}{81}y + \frac{10}{81}z + \frac{20}{81}, z' = \frac{10}{81}x + \frac{10}{81}y + \frac{77}{81}z - \frac{8}{81}]$

3. $\mathcal{P} : x - 2y + 4z + 5 = 0$, $k = 5$.

Ответ: $[x' = \frac{101}{105}x + \frac{8}{105}y - \frac{16}{105}z - \frac{4}{21}, y' = \frac{8}{105}x + \frac{89}{105}y + \frac{32}{105}z + \frac{8}{21}, z' = -\frac{16}{105}x + \frac{32}{105}y + \frac{41}{105}z - \frac{16}{21}]$

4. $\mathcal{P} : 4x - y - z + 5 = 0$, $k = 5$.

Ответ: $[x' = \frac{13}{45}x + \frac{8}{45}y + \frac{8}{45}z - \frac{8}{9}, y' = \frac{8}{45}x + \frac{43}{45}y - \frac{2}{45}z + 2/9, z' = \frac{8}{45}x - \frac{2}{45}y + \frac{43}{45}z + 2/9.]$

5. $\mathcal{P} : 3x - 3y + 2z - 5 = 0$, $k = 3$.

Ответ: $[x' = \frac{8}{11}x + 3/11y - 2/11z + \frac{5}{11}, y' = 3/11x + \frac{8}{11}y + 2/11z - \frac{5}{11}, z' = -2/11x + 2/11y + \frac{29}{33}z + \frac{10}{33}]$

6. $\mathcal{P} : 2x + y - 2z + 3 = 0$, $k = 2$.

Ответ: $[x' = \frac{7}{9}x - 1/9y + 2/9z - 1/3, y' = -1/9x + \frac{17}{18}y + 1/9z - 1/6, z' = 2/9x + 1/9y + \frac{7}{9}z + 1/3.]$

7. $\mathcal{P} : 2x + 5y + 5z + 2 = 0$, $k = 3$.

Ответ: $[x' = \frac{77}{81}x - \frac{10}{81}y - \frac{10}{81}z - \frac{4}{81}, y' = -\frac{10}{81}x + \frac{56}{81}y - \frac{25}{81}z - \frac{10}{81}, z' = -\frac{10}{81}x - \frac{25}{81}y + \frac{56}{81}z - \frac{10}{81}]$

8. $\mathcal{P} : 3x - 4y - z - 5 = 0, k = 2.$

Ответ: $[x' = \frac{43}{52}x + \frac{3}{13}y + \frac{3}{52}z + \frac{15}{52}, y' = \frac{3}{13}x + \frac{9}{13}y - \frac{1}{13}z - \frac{5}{13}, z' = \frac{3}{52}x - \frac{1}{13}y + \frac{51}{52}z - \frac{5}{52}]$

9. $\mathcal{P} : x - 5y - z - 5 = 0, k = 5.$

Ответ: $[x' = \frac{131}{135}x + \frac{4}{27}y + \frac{4}{135}z + \frac{4}{27}, y' = \frac{4}{27}x + \frac{7}{27}y - \frac{4}{27}z - \frac{20}{27}, z' = \frac{4}{135}x - \frac{4}{27}y + \frac{131}{135}z - \frac{4}{27}]$

10. $\mathcal{P} : 5x + y - 4z + 4 = 0, k = 4.$

Ответ: $[x' = \frac{31}{56}x - \frac{5}{56}y + \frac{5}{14}z - \frac{5}{14}, y' = -\frac{5}{56}x + \frac{55}{56}y + \frac{1}{14}z - \frac{1}{14}, z' = \frac{5}{14}x + \frac{1}{14}y + \frac{5}{7}z + \frac{2}{7}]$

11. $\mathcal{P} : 3x + 5y - 2z + 2 = 0, k = 5.$

Ответ: $[x' = \frac{77}{95}x - \frac{6}{19}y + \frac{12}{95}z - \frac{12}{95}, y' = -\frac{6}{19}x + \frac{9}{19}y + \frac{4}{19}z - \frac{4}{19}, z' = \frac{12}{95}x + \frac{4}{19}y + \frac{87}{95}z + \frac{8}{95}]$

12. $\mathcal{P} : 4x - 4y + 2z - 3 = 0, k = 4.$

Ответ: $[x' = \frac{2}{3}x + \frac{1}{3}y - \frac{1}{6}z + \frac{1}{4}, y' = \frac{1}{3}x + \frac{2}{3}y + \frac{1}{6}z - \frac{1}{4}, z' = -\frac{1}{6}x + \frac{1}{6}y + \frac{11}{12}z + \frac{1}{8}]$

13. $\mathcal{P} : 3x + 3y + 2z - 1 = 0, k = 4.$

Ответ: $[x' = \frac{61}{88}x - \frac{27}{88}y - \frac{9}{44}z + \frac{9}{88}, y' = -\frac{27}{88}x + \frac{61}{88}y - \frac{9}{44}z + \frac{9}{88}, z' = -\frac{9}{44}x - \frac{9}{44}y + \frac{19}{22}z + \frac{3}{44}]$

14. $\mathcal{P} : 5x - 2y - z - 3 = 0, k = 2.$

Ответ: $[x' = \frac{7}{12}x + \frac{1}{6}y + \frac{1}{12}z + \frac{1}{4}, y' = \frac{1}{6}x + \frac{14}{15}y - \frac{1}{30}z - \frac{1}{10}, z' = \frac{1}{12}x - \frac{1}{30}y + \frac{59}{60}z - \frac{1}{20}]$

15. $\mathcal{P} : 2x + 3y + 4z + 3 = 0, k = 2.$

Ответ: $[x' = \frac{27}{29}x - \frac{3}{29}y - \frac{4}{29}z - \frac{3}{29}, y' = -\frac{3}{29}x + \frac{49}{58}y - \frac{6}{29}z - \frac{9}{58}, z' = -\frac{4}{29}x - \frac{6}{29}y + \frac{21}{29}z - \frac{6}{29}]$

16. $\mathcal{P} : x + 5y + z - 2 = 0, k = 5.$

Ответ: $[x' = \frac{131}{135}x - \frac{4}{27}y - \frac{4}{135}z + \frac{8}{135}, y' = -\frac{4}{27}x + \frac{7}{27}y - \frac{4}{27}z + \frac{8}{27}, z' = -\frac{4}{135}x - \frac{4}{27}y + \frac{131}{135}z + \frac{8}{135}]$

17. $\mathcal{P} : x + 5y + 2z + 5 = 0, k = 2.$

Ответ: $[x' = \frac{59}{60}x - \frac{1}{12}y - \frac{1}{30}z - \frac{1}{12}, y' = -\frac{1}{12}x + \frac{7}{12}y - \frac{1}{6}z - \frac{5}{12}, z' = -\frac{1}{30}x - \frac{1}{6}y + \frac{14}{15}z - \frac{1}{6}]$

18. $\mathcal{P} : 2x - 3y - 4z - 1 = 0, k = 3.$

Ответ: $[x' = \frac{79}{87}x + \frac{4}{29}y + \frac{16}{87}z + \frac{4}{87}, y' = \frac{4}{29}x + \frac{23}{29}y - \frac{8}{29}z - \frac{2}{29}, z' = \frac{16}{87}x - \frac{8}{29}y + \frac{55}{87}z - \frac{8}{87}]$

19. $\mathcal{P} : 3x + 4y + 5z + 3 = 0, k = 4.$

Ответ: $[x' = \frac{173}{200}x - \frac{9}{50}y - \frac{9}{40}z - \frac{27}{200}, y' = -\frac{9}{50}x + \frac{19}{25}y - \frac{3}{10}z - \frac{9}{50}, z' = -\frac{9}{40}x - \frac{3}{10}y + \frac{5}{8}z - \frac{9}{40}]$

20. $\mathcal{P} : 3x + 3y - 2z - 2 = 0, k = 5.$

Ответ: $[x' = \frac{37}{55}x - \frac{18}{55}y + \frac{12}{55}z + \frac{12}{55}, y' = -\frac{18}{55}x + \frac{37}{55}y + \frac{12}{55}z + \frac{12}{55}, z' = \frac{12}{55}x + \frac{12}{55}y + \frac{47}{55}z - \frac{8}{55}]$

21. $\mathcal{P} : 4x - y + z + 4 = 0, k = 2.$

Ответ: $[x' = \frac{5}{9}x + \frac{1}{9}y - \frac{1}{9}z - \frac{4}{9}, y' = \frac{1}{9}x + \frac{35}{36}y + \frac{1}{36}z + \frac{1}{9}, z' = -\frac{1}{9}x + \frac{1}{36}y + \frac{35}{36}z - \frac{1}{9}]$

22. $\mathcal{P} : 3x - 4y + 5z + 5 = 0, k = 2.$

Ответ: $[x' = \frac{91}{100}x + \frac{3}{25}y - \frac{3}{20}z - \frac{3}{20}, y' = \frac{3}{25}x + \frac{21}{25}y + \frac{1}{5}z + \frac{1}{5}, z' = -\frac{3}{20}x + \frac{1}{5}y + \frac{3}{4}z - \frac{1}{4}]$

23. $\mathcal{P} : 4x - 3y + 5z - 3 = 0, k = 3.$

Ответ: $[x' = \frac{59}{75}x + \frac{4}{25}y - \frac{4}{15}z + \frac{4}{25}, y' = \frac{4}{25}x + \frac{22}{25}y + \frac{1}{5}z - \frac{3}{25}, z' = -\frac{4}{15}x + \frac{1}{5}y + \frac{2}{3}z + \frac{1}{5}]$

24. $\mathcal{P} : 2x - 4y + z + 5 = 0, k = 4.$

Ответ: $[x' = \frac{6}{7}x + \frac{2}{7}y - \frac{1}{14}z - \frac{5}{14}, y' = \frac{2}{7}x + \frac{3}{7}y + \frac{1}{7}z + \frac{5}{7}, z' = -\frac{1}{14}x + \frac{1}{7}y + \frac{27}{28}z - \frac{5}{28}]$

25. $\mathcal{P} : 4x - 4y - z + 5 = 0, k = 3.$

Ответ: $[x' = \frac{67}{99}x + \frac{32}{99}y + \frac{8}{99}z - \frac{40}{99}, y' = \frac{32}{99}x + \frac{67}{99}y - \frac{8}{99}z + \frac{40}{99}, z' = \frac{8}{99}x - \frac{8}{99}y + \frac{97}{99}z + \frac{10}{99}]$

26. $\mathcal{P} : x + 3y + 3z - 4 = 0, k = 2.$

Ответ: $[x' = \frac{37}{38}x - \frac{3}{38}y - \frac{3}{38}z + \frac{2}{19}, y' = -\frac{3}{38}x + \frac{29}{38}y - \frac{9}{38}z + \frac{6}{19}, z' = -\frac{3}{38}x - \frac{9}{38}y + \frac{29}{38}z + \frac{6}{19}]$

27. $\mathcal{P} : 4x - y + 3z - 2 = 0, k = 2.$

Ответ: $[x' = \frac{9}{13}x + \frac{1}{13}y - \frac{3}{13}z + \frac{2}{13}, y' = \frac{1}{13}x + \frac{51}{52}y + \frac{3}{52}z - \frac{1}{26}, z' = -\frac{3}{13}x + \frac{3}{52}y + \frac{43}{52}z + \frac{3}{26}]$

28. $\mathcal{P} : x - 5y + 5z + 2 = 0, k = 5.$

Ответ: $[x' = \frac{251}{255}x + \frac{4}{51}y - \frac{4}{51}z - \frac{8}{255}, y' = \frac{4}{51}x + \frac{31}{51}y + \frac{20}{51}z + \frac{8}{51}, z' = -\frac{4}{51}x + \frac{20}{51}y + \frac{31}{51}z - \frac{8}{51}]$

29. $\mathcal{P} : 2x + 4y + 4z - 5 = 0, k = 2.$

Ответ: $[x' = \frac{17}{18}x - 1/9y - 1/9z + \frac{5}{36}, y' = -1/9x + \frac{7}{9}y - 2/9z + \frac{5}{18}, z' = -1/9x - 2/9y + \frac{7}{9}z + \frac{5}{18}]$

30. $\mathcal{P} : 4x + 5y + 3z + 4 = 0, k = 3.$

Ответ: $[x' = \frac{59}{75}x - \frac{4}{15}y - \frac{4}{25}z - \frac{16}{75}, y' = -\frac{4}{15}x + 2/3y - 1/5z - \frac{4}{15}, z' = -\frac{4}{25}x - 1/5y + \frac{22}{25}z - \frac{4}{25}]$

Задача 3.3. Найти движение прямой, переводящее точку M в точку M' , если это движение является: а) собственным, б) несобственным.

1. $M(2), M'(-3).$

Ответ: $x' = x - 5, x' = -x - 1.$

7. $M(-5), M'(-2).$

Ответ: $x' = x + 3, x' = -x - 7.$

2. $M(-5), M'(2).$

Ответ: $x' = x + 7, x' = -x - 3.$

8. $M(5), M'(3).$

Ответ: $x' = x - 2, x' = -x + 8.$

3. $M(-1), M'(-3).$

Ответ: $x' = x - 2, x' = -x - 4.$

9. $M(-5), M'(1).$

Ответ: $x' = x + 6, x' = -x - 4.$

4. $M(-5), M'(3).$

Ответ: $x' = x + 8, x' = -x - 2.$

10. $M(5), M'(-3).$

Ответ: $x' = x - 8, x' = -x + 2.$

5. $M(-2), M'(-1).$

Ответ: $x' = x + 1, x' = -x - 3.$

11. $M(-3), M'(1).$

Ответ: $x' = x + 4, x' = -x - 2.$

6. $M(-5), M'(-1).$

Ответ: $x' = x + 4, x' = -x - 6.$

12. $M(1), M'(-3).$

Ответ: $x' = x - 4, x' = -x - 2.$

13. $M(3), M'(-2)$.

Ответ: $x' = x - 5, x' = -x + 1$.

14. $M(-4), M'(1)$.

Ответ: $x' = x + 5, x' = -x - 3$.

15. $M(-3), M'(-1)$.

Ответ: $x' = x + 2, x' = -x - 4$.

16. $M(-4), M'(2)$.

Ответ: $x' = x + 6, x' = -x - 2$.

17. $M(4), M'(3)$.

Ответ: $x' = x - 1, x' = -x + 7$.

18. $M(-2), M'(-3)$.

Ответ: $x' = x - 1, x' = -x - 5$.

19. $M(-2), M'(-4)$.

Ответ: $x' = x - 2, x' = -x - 6$.

20. $M(-2), M'(4)$.

Ответ: $x' = x + 6, x' = -x + 2$.

21. $M(4), M'(2)$.

Ответ: $x' = x - 2, x' = -x + 6$.

22. $M(2), M'(-1)$.

Ответ: $x' = x - 3, x' = -x + 1$.

23. $M(-1), M'(-4)$.

Ответ: $x' = x - 3, x' = -x - 5$.

24. $M(4), M'(-1)$.

Ответ: $x' = x - 5, x' = -x + 3$.

25. $M(-1), M'(4)$.

Ответ: $x' = x + 5, x' = -x + 3$.

26. $M(1), M'(3)$.

Ответ: $x' = x + 2, x' = -x + 4$.

27. $M(-2), M'(1)$.

Ответ: $x' = x + 3, x' = -x - 1$.

28. $M(1), M'(-2)$.

Ответ: $x' = x - 3, x' = -x - 1$.

29. $M(4), M'(-2)$.

Ответ: $x' = x - 6, x' = -x + 2$.

30. $M(3), M'(-1)$.

Ответ: $x' = x - 4, x' = -x + 2$.

Задача 3.4. Найти аффинное преобразование, являющееся симметрией плоскости в прямой ℓ . Система координат прямоугольная.

1. $\ell: x + 5y + 1 = 0$.
Ответ: $x' = \frac{12}{13}x - \frac{5}{13}y - 1/13$, $y' = -\frac{5}{13}x - \frac{12}{13}y - \frac{5}{13}$
2. $\ell: 5x + 4y + 2 = 0$.
Ответ: $x' = -\frac{9}{41}x - \frac{40}{41}y - \frac{20}{41}$, $y' = -\frac{40}{41}x + \frac{9}{41}y - \frac{16}{41}$
3. $\ell: x - 3y + 5 = 0$.
Ответ: $x' = 4/5x + 3/5y - 1$, $y' = 3/5x - 4/5y + 3$
4. $\ell: 2x + 4y - 5 = 0$.
Ответ: $x' = 3/5x - 4/5y + 1$, $y' = -4/5x - 3/5y + 2$
5. $\ell: x - 2y + 1 = 0$.
Ответ: $x' = 3/5x + 4/5y - 2/5$, $y' = 4/5x - 3/5y + 4/5$
6. $\ell: 3x - 4y - 2 = 0$.
Ответ: $x' = \frac{7}{25}x + \frac{24}{25}y + \frac{12}{25}$, $y' = \frac{24}{25}x - \frac{7}{25}y - \frac{16}{25}$
7. $\ell: x + 2y - 1 = 0$.
Ответ: $x' = 3/5x - 4/5y + 2/5$, $y' = -4/5x - 3/5y + 4/5$
8. $\ell: 5x + 3y - 1 = 0$.
Ответ: $x' = -\frac{8}{17}x - \frac{15}{17}y + \frac{5}{17}$, $y' = -\frac{15}{17}x + \frac{8}{17}y + \frac{3}{17}$
9. $\ell: 4x - 5y + 4 = 0$.
Ответ: $x' = \frac{9}{41}x + \frac{40}{41}y - \frac{32}{41}$, $y' = \frac{40}{41}x - \frac{9}{41}y + \frac{40}{41}$
10. $\ell: 2x - y + 5 = 0$.
Ответ: $x' = -3/5x + 4/5y - 4$, $y' = 4/5x + 3/5y + 2$
11. $\ell: x - 5y + 3 = 0$.
Ответ: $x' = \frac{12}{13}x + \frac{5}{13}y - 3/13$, $y' = \frac{5}{13}x - \frac{12}{13}y + \frac{15}{13}$
12. $\ell: x - 4y - 2 = 0$.
Ответ: $x' = \frac{15}{17}x + \frac{8}{17}y + \frac{4}{17}$, $y' = \frac{8}{17}x - \frac{15}{17}y - \frac{16}{17}$
13. $\ell: 4x - 3y + 3 = 0$.
Ответ: $x' = -\frac{7}{25}x + \frac{24}{25}y - \frac{24}{25}$, $y' = \frac{24}{25}x + \frac{7}{25}y + \frac{18}{25}$
14. $\ell: 3x - y + 3 = 0$.
Ответ: $x' = -4/5x + 3/5y - 9/5$, $y' = 3/5x + 4/5y + 3/5$
15. $\ell: x + 5y - 5 = 0$.
Ответ: $x' = \frac{12}{13}x - \frac{5}{13}y + \frac{5}{13}$, $y' = -\frac{5}{13}x - \frac{12}{13}y + \frac{25}{13}$
16. $\ell: 4x + 5y + 1 = 0$.
Ответ: $x' = \frac{9}{41}x - \frac{40}{41}y - \frac{8}{41}$, $y' = -\frac{40}{41}x - \frac{9}{41}y - \frac{10}{41}$
17. $\ell: 2x - 5y - 4 = 0$.
Ответ: $x' = \frac{21}{29}x + \frac{20}{29}y + \frac{16}{29}$, $y' = \frac{20}{29}x - \frac{21}{29}y - \frac{40}{29}$
18. $\ell: 3x + 5y - 5 = 0$.
Ответ: $x' = \frac{8}{17}x - \frac{15}{17}y + \frac{15}{17}$, $y' = -\frac{15}{17}x - \frac{8}{17}y + \frac{25}{17}$
19. $\ell: 3x - 2y + 4 = 0$.
Ответ: $x' = -\frac{5}{13}x + \frac{12}{13}y - \frac{24}{13}$, $y' = \frac{12}{13}x + \frac{5}{13}y + \frac{16}{13}$

20. $\ell: 4x + y - 2 = 0.$

Ответ: $x' = -\frac{15}{17}x - \frac{8}{17}y + \frac{16}{17}, y' = -\frac{8}{17}x + \frac{15}{17}y + \frac{4}{17}$

21. $\ell: 3x - 2y - 5 = 0.$

Ответ: $x' = -\frac{5}{13}x + \frac{12}{13}y + \frac{30}{13}, y' = \frac{12}{13}x + \frac{5}{13}y - \frac{20}{13}$

22. $\ell: 5x - 3y - 4 = 0.$

Ответ: $x' = -\frac{8}{17}x + \frac{15}{17}y + \frac{20}{17}, y' = \frac{15}{17}x + \frac{8}{17}y - \frac{12}{17}$

23. $\ell: 4x + 3y - 5 = 0.$

Ответ: $x' = -\frac{7}{25}x - \frac{24}{25}y + \frac{8}{5}, y' = -\frac{24}{25}x + \frac{7}{25}y + \frac{6}{5}$

24. $\ell: 4x - 5y + 1 = 0.$

Ответ: $x' = \frac{9}{41}x + \frac{40}{41}y - \frac{8}{41}, y' = \frac{40}{41}x - \frac{9}{41}y + \frac{10}{41}$

25. $\ell: 4x + y - 1 = 0.$

Ответ: $x' = -\frac{15}{17}x - \frac{8}{17}y + \frac{8}{17}, y' = -\frac{8}{17}x + \frac{15}{17}y + \frac{2}{17}$

26. $\ell: x + 2y + 3 = 0.$

Ответ: $x' = \frac{3}{5}x - \frac{4}{5}y - \frac{6}{5}, y' = -\frac{4}{5}x - \frac{3}{5}y - \frac{12}{5}$

27. $\ell: 5x + 4y + 5 = 0.$

Ответ: $x' = -\frac{9}{41}x - \frac{40}{41}y - \frac{50}{41}, y' = -\frac{40}{41}x + \frac{9}{41}y - \frac{40}{41}$

28. $\ell: 2x + 4y + 2 = 0.$

Ответ: $x' = \frac{3}{5}x - \frac{4}{5}y - \frac{2}{5}, y' = -\frac{4}{5}x - \frac{3}{5}y - \frac{4}{5}$

29. $\ell: 5x - y + 5 = 0.$

Ответ: $x' = -\frac{12}{13}x + \frac{5}{13}y - \frac{25}{13}, y' = \frac{5}{13}x + \frac{12}{13}y + \frac{5}{13}$

30. $\ell: x + 3y - 2 = 0.$

Ответ: $x' = \frac{4}{5}x - \frac{3}{5}y + \frac{2}{5}, y' = -\frac{3}{5}x - \frac{4}{5}y + \frac{6}{5}$

Задача 3.5. Определить, является ли движение плоскости $(x, y) \mapsto (x', y')$ собственным или несобственным. Если преобразование собственное, то найти неподвижную точку и угол поворота, если несобственное, то найти инвариантную прямую и вектор сдвига.

1. а) $x' = \frac{15}{17}x + \frac{8}{17}y - \frac{24}{17}, y' = -\frac{8}{17}x + \frac{15}{17}y + \frac{40}{17};$

б) $x' = \frac{3}{5}x + \frac{4}{5}y + \frac{16}{5}, y' = \frac{4}{5}x - \frac{3}{5}y - \frac{7}{5}.$

Ответ: а) $O(4, 4), \phi = -\arctan\left(\frac{8}{15}\right);$ б) $\ell: x - 2y - 3 = 0, \vec{v} = (2, 1).$

2. а) $x' = \frac{24}{25}x + \frac{7}{25}y - \frac{9}{25}, y' = -\frac{7}{25}x + \frac{24}{25}y + \frac{37}{25};$

б) $x' = \frac{3}{5}x + \frac{4}{5}y - 6, y' = \frac{4}{5}x - \frac{3}{5}y + 2.$

Ответ: а) $O(5, 2), \phi = -\arctan\left(\frac{7}{24}\right);$ б) $\ell: x - 2y + 5 = 0, \vec{v} = (-4, -2).$

$$3. a) x' = \frac{8}{17}x - \frac{15}{17}y + \frac{94}{17}, y' = -\frac{15}{17}x - \frac{8}{17}y - \frac{36}{17};$$

$$b) x' = -\frac{3}{5}x + \frac{4}{5}y - \frac{12}{5}, y' = -\frac{4}{5}x - \frac{3}{5}y - \frac{56}{5}.$$

Ответ: а) $\ell : 3x + 5y - 3 = 0, \vec{v} = (5, -3)$, б) $O(-4, -5), \phi = \arctan(4/3) - \pi$.

$$4. a) x' = -\frac{7}{25}x - \frac{24}{25}y + \frac{83}{25}, y' = -\frac{24}{25}x + \frac{7}{25}y - \frac{94}{25};$$

$$b) x' = -\frac{5}{13}x + \frac{12}{13}y - \frac{66}{13}, y' = -\frac{12}{13}x - \frac{5}{13}y - \frac{96}{13}.$$

Ответ: а) $\ell : 4x + 3y - 1 = 0, \vec{v} = (3, -4)$, б) $O(-5, -2), \phi = \arctan(\frac{12}{5}) - \pi$.

$$5. a) x' = \frac{3}{5}x + \frac{4}{5}y + \frac{22}{5}, y' = \frac{4}{5}x - \frac{3}{5}y + \frac{6}{5};$$

$$b) x' = -\frac{8}{17}x + \frac{15}{17}y + \frac{35}{17}, y' = -\frac{15}{17}x - \frac{8}{17}y + \frac{55}{17}.$$

Ответ: а) $\ell : x - 2y - 1 = 0, \vec{v} = (4, 2)$, б) $O(2, 1), \phi = \arctan(\frac{15}{8}) - \pi$.

$$6. a) x' = -\frac{7}{25}x - \frac{24}{25}y + \frac{99}{25}, y' = -\frac{24}{25}x + \frac{7}{25}y - \frac{82}{25};$$

$$b) x' = \frac{3}{5}x - \frac{4}{5}y - \frac{12}{5}, y' = \frac{4}{5}x + \frac{3}{5}y - \frac{16}{5}.$$

Ответ: а) $\ell : 4x + 3y - 3 = 0, \vec{v} = (3, -4)$, б) $O(2, -4), \phi = \arctan(4/3)$.

$$7. a) x' = \frac{5}{13}x - \frac{12}{13}y - 4, y' = \frac{12}{13}x + \frac{5}{13}y - 4;$$

$$b) x' = \frac{9}{41}x - \frac{40}{41}y + \frac{402}{41}, y' = -\frac{40}{41}x - \frac{9}{41}y - \frac{338}{41}.$$

Ответ: а) $O(1, -5), \phi = \arctan(\frac{12}{5})$; б) $\ell : 4x + 5y + 1 = 0, \vec{v} = (10, -8)$.

$$8. a) x' = \frac{8}{17}x - \frac{15}{17}y - \frac{3}{17}, y' = \frac{15}{17}x + \frac{8}{17}y + \frac{39}{17};$$

$$b) x' = -\frac{5}{13}x + \frac{12}{13}y + \frac{58}{13}, y' = \frac{12}{13}x + \frac{5}{13}y + \frac{74}{13}.$$

Ответ: а) $O(-2, 1), \phi = \arctan(\frac{15}{8})$; б) $\ell : 3x - 2y - 1 = 0, \vec{v} = (4, 6)$.

$$9. a) x' = \frac{7}{25}x - \frac{24}{25}y + \frac{162}{25}, y' = \frac{24}{25}x + \frac{7}{25}y - \frac{66}{25};$$

$$b) x' = -\frac{8}{17}x + \frac{15}{17}y + \frac{117}{17}, y' = \frac{15}{17}x + \frac{8}{17}y + \frac{161}{17}.$$

Ответ: а) $O(5, 3), \phi = \arctan(\frac{24}{7})$; б) $\ell : 5x - 3y - 3 = 0, \vec{v} = (6, 10)$.

$$10. a) x' = -\frac{3}{5}x - \frac{4}{5}y + 3, y' = -\frac{4}{5}x + \frac{3}{5}y + 4;$$

$$b) x' = \frac{4}{5}x - \frac{3}{5}y + \frac{6}{5}, y' = \frac{3}{5}x + \frac{4}{5}y + \frac{12}{5}.$$

ОТВЕТ: а) $\ell : 2x + y - 5 = 0$, $\vec{v} = (-1, 2)$, б) $O(-3, 3)$, $\phi = \arctan(3/4)$.

11. а) $x' = \frac{9}{41}x - \frac{40}{41}y - \frac{426}{41}$, $y' = -\frac{40}{41}x - \frac{9}{41}y + \frac{308}{41}$;

б) $x' = \frac{12}{13}x - \frac{5}{13}y + \frac{8}{13}$, $y' = \frac{5}{13}x + \frac{12}{13}y - \frac{14}{13}$.

ОТВЕТ: а) $\ell : 4x + 5y + 2 = 0$, $\vec{v} = (-10, 8)$, б) $O(3, 1)$, $\phi = \arctan(\frac{5}{12})$.

12. а) $x' = -\frac{12}{13}x - \frac{5}{13}y - \frac{12}{13}$, $y' = -\frac{5}{13}x + \frac{12}{13}y - \frac{70}{13}$;

б) $x' = \frac{15}{17}x - \frac{8}{17}y - \frac{14}{17}$, $y' = \frac{8}{17}x + \frac{15}{17}y - \frac{46}{17}$.

ОТВЕТ: а) $\ell : 5x + y + 5 = 0$, $\vec{v} = (1, -5)$, б) $O(5, -3)$, $\phi = \arctan(\frac{8}{15})$.

13. а) $x' = \frac{24}{25}x - \frac{7}{25}y + 1$, $y' = \frac{7}{25}x + \frac{24}{25}y - 1$;

б) $x' = \frac{3}{5}x + \frac{4}{5}y - 3$, $y' = \frac{4}{5}x - \frac{3}{5}y - 4$.

ОТВЕТ: а) $O(4, 3)$, $\phi = \arctan(\frac{7}{24})$; б) $\ell : 2x - 4y - 5 = 0$, $\vec{v} = (-4, -2)$.

14. а) $x' = \frac{3}{5}x + \frac{4}{5}y + 2$, $y' = -\frac{4}{5}x + \frac{3}{5}y - 2$;

б) $x' = \frac{3}{5}x - \frac{4}{5}y + \frac{21}{5}$, $y' = -\frac{4}{5}x - \frac{3}{5}y - 8/5$.

ОТВЕТ: а) $O(-1, -3)$, $\phi = -\arctan(4/3)$; б) $\ell : 2x + 4y - 1 = 0$, $\vec{v} = (4, -2)$.

15. а) $x' = \frac{7}{25}x + \frac{24}{25}y - \frac{138}{25}$, $y' = -\frac{24}{25}x + \frac{7}{25}y - \frac{84}{25}$;

б) $x' = -\frac{3}{5}x - \frac{4}{5}y + \frac{22}{5}$, $y' = -\frac{4}{5}x + \frac{3}{5}y - \frac{14}{5}$.

ОТВЕТ: а) $O(-5, 2)$, $\phi = -\arctan(\frac{24}{7})$; б) $\ell : 2x + y - 3 = 0$, $\vec{v} = (2, -4)$.

16. а) $x' = \frac{20}{29}x + \frac{21}{29}y + 3$, $y' = -\frac{21}{29}x + \frac{20}{29}y - 3$;

б) $x' = \frac{3}{5}x - \frac{4}{5}y + 6$, $y' = -\frac{4}{5}x - \frac{3}{5}y + 2$.

ОТВЕТ: а) $O(-2, -5)$, $\phi = -\arctan(\frac{21}{20})$; б) $\ell : x + 2y - 5 = 0$, $\vec{v} = (4, -2)$.

17. а) $x' = \frac{4}{5}x + \frac{3}{5}y - 1$, $y' = -\frac{3}{5}x + \frac{4}{5}y - 1$;

б) $x' = \frac{4}{5}x + \frac{3}{5}y - 5$, $y' = \frac{3}{5}x - \frac{4}{5}y - 5$.

ОТВЕТ: а) $O(-2, 1)$, $\phi = -\arctan(3/4)$; б) $\ell : x - 3y - 5 = 0$, $\vec{v} = (-6, -2)$.

18. a) $x' = \frac{21}{29}x + \frac{20}{29}y + \frac{165}{29}$, $y' = \frac{20}{29}x - \frac{21}{29}y + \frac{8}{29}$;
 b) $x' = \frac{12}{13}x + \frac{5}{13}y + \frac{24}{13}$, $y' = -\frac{5}{13}x + \frac{12}{13}y + \frac{16}{13}$.

Ответ: a) $\ell : 2x - 5y - 5 = 0$, $\vec{v} = (5, 2)$, b) $O(4, -4)$, $\phi = -\arctan\left(\frac{5}{12}\right)$.

19. a) $x' = \frac{7}{25}x + \frac{24}{25}y - \frac{212}{25}$, $y' = \frac{24}{25}x - \frac{7}{25}y - \frac{134}{25}$;
 b) $x' = \frac{15}{17}x + \frac{8}{17}y - \frac{16}{17}$, $y' = -\frac{8}{17}x + \frac{15}{17}y + \frac{38}{17}$.

Ответ: a) $\ell : 3x - 4y + 2 = 0$, $\vec{v} = (-8, -6)$, b) $O(4, 3)$, $\phi = -\arctan\left(\frac{8}{15}\right)$.

20. a) $x' = \frac{24}{25}x + \frac{7}{25}y - \frac{3}{25}$, $y' = -\frac{7}{25}x + \frac{24}{25}y + \frac{29}{25}$;
 b) $x' = \frac{3}{5}x + \frac{4}{5}y + \frac{14}{5}$, $y' = \frac{4}{5}x - \frac{3}{5}y + \frac{22}{5}$.

Ответ: a) $O(4, 1)$, $\phi = -\arctan\left(\frac{7}{24}\right)$; b) $\ell : x - 2y + 3 = 0$, $\vec{v} = (4, 2)$.

21. a) $x' = \frac{7}{25}x + \frac{24}{25}y + \frac{218}{25}$, $y' = \frac{24}{25}x - \frac{7}{25}y + \frac{126}{25}$;
 b) $x' = -\frac{3}{5}x + \frac{4}{5}y + \frac{32}{5}$, $y' = -\frac{4}{5}x - \frac{3}{5}y - \frac{4}{5}$.

Ответ: a) $\ell : 3x - 4y - 3 = 0$, $\vec{v} = (8, 6)$, b) $O(3, -2)$, $\phi = \arctan(4/3) - \pi$.

22. a) $x' = -\frac{7}{25}x + \frac{24}{25}y - 8$, $y' = -\frac{24}{25}x - \frac{7}{25}y$;
 b) $x' = -\frac{15}{17}x - \frac{8}{17}y - \frac{57}{17}$, $y' = -\frac{8}{17}x + \frac{15}{17}y + \frac{58}{17}$.

Ответ: a) $O(-4, 3)$, $\phi = \arctan\left(\frac{24}{7}\right) - \pi$; b) $\ell : 4x + y + 5 = 0$, $\vec{v} = (-1, 4)$.

23. a) $x' = -\frac{20}{29}x + \frac{21}{29}y - \frac{7}{29}$, $y' = -\frac{21}{29}x - \frac{20}{29}y + \frac{287}{29}$;
 b) $x' = -\frac{5}{13}x - \frac{12}{13}y + \frac{76}{13}$, $y' = -\frac{12}{13}x + \frac{5}{13}y - \frac{62}{13}$.

Ответ: a) $O(2, 5)$, $\phi = \arctan\left(\frac{21}{20}\right) - \pi$; b) $\ell : 3x + 2y - 4 = 0$, $\vec{v} = (4, -6)$.

24. a) $x' = \frac{9}{41}x + \frac{40}{41}y - \frac{173}{41}$, $y' = \frac{40}{41}x - \frac{9}{41}y - \frac{204}{41}$;
 b) $x' = -\frac{4}{5}x + \frac{3}{5}y + \frac{27}{5}$, $y' = -\frac{3}{5}x - \frac{4}{5}y - \frac{21}{5}$.

Ответ: a) $\ell : 4x - 5y - 4 = 0$, $\vec{v} = (-5, -4)$, b) $O(2, -3)$, $\phi = \arctan(3/4) - \pi$.

25. a) $x' = -\frac{12}{13}x + \frac{5}{13}y - \frac{30}{13}$, $y' = -\frac{5}{13}x - \frac{12}{13}y - \frac{110}{13}$;
 b) $x' = -\frac{8}{17}x - \frac{15}{17}y - \frac{117}{17}$, $y' = -\frac{15}{17}x + \frac{8}{17}y + \frac{161}{17}$.

Ответ: а) $O(-2, -4)$, $\phi = \arctan\left(\frac{5}{12}\right) - \pi$; б) $\ell : 5x + 3y + 3 = 0$, $\vec{v} = (-6, 10)$.

26. а) $x' = -\frac{24}{25}x + \frac{7}{25}y - \frac{168}{25}$, $y' = -\frac{7}{25}x - \frac{24}{25}y - \frac{224}{25}$;
 б) $x' = -\frac{3}{5}x - \frac{4}{5}y + 6$, $y' = -\frac{4}{5}x + \frac{3}{5}y - 2$.

Ответ: а) $O(-4, -4)$, $\phi = \arctan\left(\frac{7}{24}\right) - \pi$; б) $\ell : 2x + y - 5 = 0$, $\vec{v} = (2, -4)$.

27. а) $x' = -\frac{12}{13}x - \frac{5}{13}y - \frac{7}{13}$, $y' = -\frac{5}{13}x + \frac{12}{13}y - \frac{69}{13}$;
 б) $x' = -\frac{21}{29}x + \frac{20}{29}y + \frac{170}{29}$, $y' = -\frac{20}{29}x - \frac{21}{29}y + \frac{300}{29}$.

Ответ: а) $\ell : 5x + y + 4 = 0$, $\vec{v} = (1, -5)$, б) $O(5, 4)$, $\phi = \arctan\left(\frac{20}{21}\right) - \pi$.

28. а) $x' = \frac{21}{29}x + \frac{20}{29}y - \frac{270}{29}$, $y' = \frac{20}{29}x - \frac{21}{29}y - \frac{166}{29}$;
 б) $x' = -\frac{3}{5}x - \frac{4}{5}y + \frac{24}{5}$, $y' = \frac{4}{5}x - \frac{3}{5}y - \frac{32}{5}$.

Ответ: а) $\ell : 2x - 5y - 5 = 0$, $\vec{v} = (-10, -4)$, б) $O(4, -2)$, $\phi = -\arctan(4/3) + \pi$.

29. а) $x' = -\frac{5}{13}x - \frac{12}{13}y + \frac{48}{13}$, $y' = \frac{12}{13}x - \frac{5}{13}y - \frac{6}{13}$;
 б) $x' = -\frac{8}{17}x + \frac{15}{17}y - \frac{66}{17}$, $y' = \frac{15}{17}x + \frac{8}{17}y - \frac{76}{17}$.

Ответ: а) $O(2, 1)$, $\phi = -\arctan\left(\frac{12}{5}\right) + \pi$; б) $\ell : 5x - 3y + 3 = 0$, $\vec{v} = (-3, -5)$.

30. а) $x' = -\frac{20}{29}x - \frac{21}{29}y + \frac{231}{29}$, $y' = \frac{21}{29}x - \frac{20}{29}y + \frac{133}{29}$;
 б) $x' = \frac{9}{41}x + \frac{40}{41}y - \frac{442}{41}$, $y' = \frac{40}{41}x - \frac{9}{41}y - \frac{288}{41}$.

Ответ: а) $O(3, 4)$, $\phi = -\arctan\left(\frac{21}{20}\right) + \pi$; б) $\ell : 4x - 5y + 4 = 0$, $\vec{v} = (-10, -8)$.

Задача 3.6. Доказать, что данное отображение является преобразованием подобия. Разложить его в композицию $\psi \circ \Phi \circ \theta \circ \eta$, где

- η — тождественное: $(x, y) \mapsto (x, y)$, если исходное собственное и симметрия относительно оси Ox : $(x, y) \mapsto (x, -y)$, если несобственное,
- θ — параллельный перенос на вектор $\vec{v} = (a, b)$:
 $(x, y) \mapsto (x + a, y + b)$,
- Φ — поворот на угол φ : $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$,

- ψ — гомотетия с центром в начале координат и коэффициентом k : $(x, y) \mapsto (kx, ky)$.

Система координат прямоугольная.

1. $x' = 7x + 24y, y' = -24x + 7y - 2$.

Ответ: $[sob, a = \frac{48}{625}, b = -\frac{14}{625}, \lambda = 25, \{\varphi = -\arctg(\frac{24}{7})\}]$

2. $x' = 5x - 12y, y' = 12x + 5y + 3$.

Ответ: $[sob, a = \frac{36}{169}, b = \frac{15}{169}, \lambda = 13, \{\varphi = \arctg(\frac{12}{5})\}]$

3. $x' = 20x - 21y - 1, y' = 21x + 20y - 3$.

Ответ: $[sob, a = -\frac{83}{841}, b = -\frac{39}{841}, \lambda = 29, \{\varphi = \arctg(\frac{21}{20})\}]$

4. $x' = 21x - 20y - 1, y' = 20x + 21y + 2$.

Ответ: $[sob, a = \frac{19}{841}, b = \frac{62}{841}, \lambda = 29, \{\varphi = \arctg(\frac{20}{21})\}]$

5. $x' = -20x - 21y - 3, y' = -21x + 20y - 1$.

Ответ: $[nsob, a = \frac{81}{841}, b = -\frac{43}{841}, \lambda = 29, \{\varphi = \arctg(\frac{21}{20}) - \pi\}]$

6. $x' = 8x + 15y - 2, y' = 15x - 8y - 3$.

Ответ: $[nsob, a = -\frac{61}{289}, b = \frac{6}{289}, \lambda = 17, \{\varphi = \arctg(\frac{15}{8})\}]$

7. $x' = 12x + 5y - 2, y' = 5x - 12y - 2$.

Ответ: $[nsob, a = -\frac{34}{169}, b = -\frac{14}{169}, \lambda = 13, \{\varphi = \arctg(\frac{5}{12})\}]$

8. $x' = -12x + 5y - 3, y' = -5x - 12y - 3$.

Ответ: $[sob, a = \frac{51}{169}, b = \frac{21}{169}, \lambda = 13, \{\varphi = \arctg(\frac{5}{12}) - \pi\}]$

9. $x' = -3x - 4y - 3, y' = 4x - 3y - 2$.

Ответ: $[sob, a = 1/25, b = \frac{18}{25}, \lambda = 5, \{\varphi = -\arctg(4/3) + \pi\}]$

10. $x' = 8x + 15y + 1, y' = -15x + 8y + 2$.

Ответ: $[sob, a = -\frac{22}{289}, b = \frac{31}{289}, \lambda = 17, \{\varphi = -\arctg(\frac{15}{8})\}]$

11. $x' = 4x + 3y + 1, y' = 3x - 4y - 1$.

Ответ: $[nsob, a = 1/25, b = -\frac{7}{25}, \lambda = 5, \{\varphi = \arctg(3/4)\}]$

12. $x' = 20x + 21y - 3, y' = -21x + 20y - 3$.

Ответ: $[sob, a = \frac{3}{841}, b = -\frac{123}{841}, \lambda = 29, \{\varphi = -\arctg(\frac{21}{20})\}]$

13. $x' = -24x - 7y - 1, y' = -7x + 24y + 3.$

ОТВЕТ: $[nsob, a = \frac{3}{625}, b = -\frac{79}{625}, \lambda = 25, \{\varphi = \arctg(\frac{7}{24}) - \pi\}]$

14. $x' = 24x + 7y - 2, y' = 7x - 24y - 3.$

ОТВЕТ: $[nsob, a = -\frac{69}{625}, b = -\frac{58}{625}, \lambda = 25, \{\varphi = \arctg(\frac{7}{24})\}]$

15. $x' = -8x - 15y - 1, y' = -15x + 8y - 3.$

ОТВЕТ: $[nsob, a = \frac{53}{289}, b = \frac{9}{289}, \lambda = 17, \{\varphi = \arctg(\frac{15}{8}) - \pi\}]$

16. $x' = 5x - 12y + 3, y' = -12x - 5y - 3.$

ОТВЕТ: $[nsob, a = \frac{51}{169}, b = \frac{21}{169}, \lambda = 13, \{\varphi = -\arctg(\frac{12}{5})\}]$

17. $x' = -5x + 12y, y' = 12x + 5y + 1.$

ОТВЕТ: $[nsob, a = \frac{12}{169}, b = -\frac{5}{169}, \lambda = 13, \{\varphi = -\arctg(\frac{12}{5}) + \pi\}]$

18. $x' = -7x - 24y + 2, y' = 24x - 7y + 3.$

ОТВЕТ: $[sob, a = \frac{58}{625}, b = -\frac{69}{625}, \lambda = 25, \{\varphi = -\arctg(\frac{24}{7}) + \pi\}]$

19. $x' = 21x + 20y - 1, y' = 20x - 21y + 3.$

ОТВЕТ: $[nsob, a = \frac{39}{841}, b = \frac{83}{841}, \lambda = 29, \{\varphi = \arctg(\frac{20}{21})\}]$

20. $x' = -15x - 8y - 2, y' = 8x - 15y + 1.$

ОТВЕТ: $[sob, a = \frac{38}{289}, b = \frac{1}{289}, \lambda = 17, \{\varphi = -\arctg(\frac{8}{15}) + \pi\}]$

21. $x' = 21x + 20y - 1, y' = -20x + 21y + 3.$

ОТВЕТ: $[sob, a = -\frac{81}{841}, b = \frac{43}{841}, \lambda = 29, \{\varphi = -\arctg(\frac{20}{21})\}]$

22. $x' = 24x + 7y - 2, y' = -7x + 24y.$

ОТВЕТ: $[sob, a = -\frac{48}{625}, b = -\frac{14}{625}, \lambda = 25, \{\varphi = -\arctg(\frac{7}{24})\}]$

23. $x' = -15x + 8y + 1, y' = 8x + 15y + 2.$

ОТВЕТ: $[nsob, a = \frac{1}{289}, b = -\frac{38}{289}, \lambda = 17, \{\varphi = -\arctg(\frac{8}{15}) + \pi\}]$

24. $x' = -4x - 3y + 3, y' = 3x - 4y.$

ОТВЕТ: $[sob, a = -\frac{12}{25}, b = -\frac{9}{25}, \lambda = 5, \{\varphi = -\arctg(3/4) + \pi\}]$

25. $x' = -4x + 3y + 3, y' = 3x + 4y.$

ОТВЕТ: $[nsob, a = -\frac{12}{25}, b = -\frac{9}{25}, \lambda = 5, \{\varphi = -\arctg(3/4) + \pi\}]$

26. $x' = 4x - 3y + 3, y' = 3x + 4y.$

ОТВЕТ: $[sob, a = \frac{12}{25}, b = -\frac{9}{25}, \lambda = 5, \{\varphi = \arctg(3/4)\}]$

27. $x' = 3x - 4y - 1, y' = 4x + 3y - 2.$

Ответ: $[sob, a = -\frac{11}{25}, b = -\frac{2}{25}, \lambda = 5, \{\varphi = \text{arctg}(4/3)\}]$

28. $x' = 7x + 24y + 3, y' = 24x - 7y - 3.$

Ответ: $[nsob, a = -\frac{51}{625}, b = -\frac{93}{625}, \lambda = 25, \{\varphi = \text{arctg}(\frac{24}{7})\}]$

29. $x' = 12x - 5y, y' = -5x - 12y + 1.$

Ответ: $[nsob, a = -\frac{5}{169}, b = \frac{12}{169}, \lambda = 13, \{\varphi = -\text{arctg}(\frac{5}{12})\}]$

30. $x' = -15x - 8y - 1, y' = -8x + 15y - 2.$

Ответ: $[nsob, a = \frac{31}{289}, b = \frac{22}{289}, \lambda = 17, \{\varphi = \text{arctg}(\frac{8}{15}) - \pi\}]$

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